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**QUESTION BANK**
**CE2404 – PRESTRESSED CONCRETE STRUCTURES**
**UNIT 1- INTRODUCTION – THEORY AND BEHAVIOUR**

**PART – A (2 marks)**

1. **List out the advantages of prestressed concrete. (AUC Nov/Dec 2011 & 2012)**
   - In case of fully prestressed member, which are free from tensile stresses under working loads, the cross section is more efficiently utilized when compared with a reinforced concrete section which is cracked under working loads.
   - The flexural member is stiffer under working loads than a reinforced concrete member of the same length.
2. **What is meant by pretensioned and post tensioned concrete? (AUC Nov/Dec 2010 & 2011)**

   **Pre tensioning:** A method of Pre stressing concrete in which the tendons are tensioned before the concrete is placed. In this method, the prestress is imparted to concrete by bond between steel and concrete.
   **Post tensioning:** A method of pre stressing concrete by tensioning the tendons against hardened concrete. In this method, the prestress is imparted to concrete by bearing.
3. **Why is high tensile steel needed for prestressed concrete construction? (AUC Nov/Dec 2012)**
   - High strength concrete is necessary for prestress concrete as the material offers highly resistance in tension, shear bond and bearing. In the zone of anchorage the bearing stresses being hired; high strength concrete is invariably preferred to minimizing the cost. High strength concrete is less liable to shrinkage cracks and has lighter modulus of elasticity and smaller ultimate creep strain resulting in a smaller loss of prestress in steel. The use of high strength concrete results in a reduction in a cross sectional dimensions of prestress concrete structural element with a reduced dead weight of the material longer span become technically and economically practicable.
   - Tensile strength of high tensile steel is in the range of 1400 to 2000 $N/mm^2$ and if initially stress upto 1400 $N/mm^2$ their will be still large stress in the high tensile reinforcement after making deduction for loss of prestress. Therefore high tensile steel is made for prestress concrete.
4. **What are the various methods of prestressing? (AUC May/June 2013, Apr/May 2010)**
   - Pre-tensioning
   - Post-tensioning
5. **What are the systems of prestressing? (AUC May/June 2013)**
   - Pre-tensioning system
   - Post-tensioning system

6. **List the loss of prestress.** **(AUC Nov/Dec 2010 & 2013)**

- Nature of losses of prestress.
- Loss due to elastic deformation of concrete.
- Loss due to shrinkage of concrete.
- Loss due to creep of concrete.
- Loss due to relaxation of stress in steel.
- Loss of stress due to friction.
- Loss due to anchorage slip.

7. **What are the classifications of prestressed concrete structures?** **(AUC Nov/Dec 2013)**
    - Externally or internally prestressed
    - Pretensioning and post tensioning
    - End-Anchored or Non-End Anchored Tendons
    - Bonded or unbonded tendons
    - Precast, cast-in-place, composite construction
    - Partial or full prestressing

8. **Define load balancing concept.** **(AUC Apr/May 2011 & 2012)**

   It is possible to select cable profiles in a prestressed concrete member such that the traverse component of the cable force balances the given type of external loads. This can be readily illustrated by considering the free body of concrete with the tendon replaced by forces acting on the concrete beam.

9. **What are the factors influencing deflections?** **(AUC Apr/May 2011)**
    - Length of the deflection field
    - Spacing between the deflection plate
    - Difference of potential between the plates
    - Accelerating voltage of the second anode.

10. **What are the sources of prestress force?** **(AUC Apr/May 2012)**
    - Mechanical
    - Hydraulic
    - Electrical
    - Chemical

11. **Define kern distance.** **(AUC Apr/May 2010)**

    Kern is the core area of the section in which if the load applied tension will not be induced in the section $K_t = Z_b / A$, $K_b = Z_t / A$,

    If the load applied at $K_t$ compressive stress will be the maximum at the top most fiber and zero stress will be at the bottom most fiber. If the load applied at $K_b$ compressive stress will be the maximum at the bottom most fiber and zero stress will be at the top most fiber.

12. **What is Relaxation of steel?**

    When a high tensile steel wire is stretch and maintained at a constant strain the initially force in the wire does not remain constant but decrease with time. The decrease of stress in steel at constant strain is termed relaxation of steel.

13. **What is concordant prestressing?**

    Pre stressing of members in which the cable follow a concordant profile. In case of statically indeterminate structures. It does not cause any changes in support reaction.

**14. Define bonded and non bonded prestressing concrete.**

**Bonded prestressing:** Concrete in which prestress is imparted to concrete through bond between the tendons and surrounding concrete. Pre tensioned members belong to this group.

**Non-bonded prestressing:** A method of construction in which the tendons are not bonded to the surrounding concrete. The tendons may be placed in ducts formed in the concrete members or they may be placed outside the concrete section.

**15. Define axial prestressing.**

Members in which the entire cross-section of concrete has a uniform compressive prestress. In this type of prestressing, the centroid, of the tendons coincides with that of the concrete section.

**16. Define prestressed concrete.**

It is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from external loads (or) counteracted to a desire degree in reinforced concrete member the prestress is commonly introduced by tensioning the steel reinforcement.

**17. Define anchorage.**

A device generally used to enable the tendon to impart and maintain prestress to the concrete is called anchorage. E.g. Fressinet, BBRV systems, etc.,

**18. What are the main factors for concrete used in PSC?**

- Ordinary Portland cement-based concrete is used but strength usually greater than 50 $N/mm^2$;
- A high early strength is required to enable quicker application of prestress;
- A larger elastic modulus is needed to reduce the shortening of the member;
- A mix that reduces creep of the concrete to minimize losses of prestress;

**19. What are the uses of prestressed concrete?**

- Railway Sleepers;
- Communications poles;
- Pre-tensioned precast “hollow core” slabs;
- Pre-tensioned Precast Double T units - for very long spans (e.g., 16 m span for car parks);
- Pre-tensioned precast inverted T beam for short-span bridges;
- Post-tensioned ribbed slab;
- This is “glued segmental” construction;

**20. Define Magnel diagram.**

A Magnel Diagram is a plot of the four lines associated with the limits on stress. As can be seen, when these four equations are plotted, a feasible region is found in which points of 1 *P* and *e* simultaneously satisfy all four equations. Any such point then satisfies all four stress limits.

**PART – B (16 marks)**

1. **A rectangular prestressed beam 150 mm wide and 300 mm deep is used over an effective span of 10 m. The cable with zero eccentricity at the supports and linearly varying to 50 mm at the centre carries an effective prestressing force of 500 kN. Find the magnitude of the concentrated load located at the centre of the span for the following conditions at the centre of span section:**
   a) **If the load counteracts the bending effect of the prestressing force (neglecting self weight of beam) and**
   b) **If the pressure line passes through the upper kern of the section under the action of the external load, self weight and prestress. (AUC Nov/Dec 2011, Apr/May 2010)**

**Solution:**

$$A = (150 \times 300) = 45 \times 10^3 \text{ mm}^2$$

$$Z = \left(\frac{150 \times 300^2}{6}\right) = 225 \times 10^4 \text{ mm}^3$$

Self-weight of beam, $g = (0.15 \times 0.3 \times 24) = 1.08$ kN/m

$$P = 500 \text{ kN} \quad e = 50 \text{ mm}$$

If the inclination of the cable to the horizontal is $\theta$, and $Q$ = concentrated load at the centre of the span,

for load balancing,

(a) $Q = 2P \sin\theta = 2P \tan\theta = \left(\dfrac{2 \times 500 \times 50}{5 \times 1000}\right) = 10$ kN

(b) Moment due to self-weight $= (0.125 \times 1.08 \times 10^2) = 13.5$ kN m

$$\text{Stress due to self-weight} = \left(\frac{13.5 \times 10^6}{225 \times 10^4}\right) = +6 \text{ N/mm}^2$$

$$\text{Stresses due to prestressing} = \left(\frac{P}{A} + \frac{Pe}{Z}\right) = \left[\frac{500 \times 10^3}{45 \times 10^3}\right] + \left[\frac{500 \times 10^3 \times 50}{225 \times 10^4}\right]$$

Stress at the bottom fibre $= 22.22$ N/mm$^2$

If $Q$ = concentrated load at the centre of the span,

Moment at the centre of span $= (Q \times 10)/4 = 2.5\,Q$

$$\text{Bending stress} = \left[\frac{(2.5Q) \times 10^6}{225 \times 10^4}\right]$$

If the pressure line passes through the upper kern at the section, the bottom fibre = 0. Thus,

$$\left[\frac{2.5Q \times 10^6}{225 \times 10^4}\right] + 6 = 22.22$$

$$Q = 14.60 \text{ kN.}$$

**2. Describe the effect of loading on the tensile stresses in tendons. (AUC Nov/Dec 2011)**

A prestressed member undergoes deformation due to the action of the prestressing force and transverse loads acting on the member. Consequently, the curvature of the cable changes, which results in a slight variation of stresses in the tendons. Considering Fig. 4.21, in which a concrete beam of span $L$ is prestressed by a cable carrying an effective force $P$ at an eccentricity, $e$ the rotation $\theta_p$ at the supports due to hogging of the beam is obtained by applying Mohr's theorem as,

$$\theta_p = \left(\frac{\text{Area of bending moment diagram}}{\text{Flexural rigidity}}\right) = \left(\frac{PeL}{2EI}\right)$$

where $EI$ = flexural rigidity of the beam.

If the beam supports a total uniformly distributed load of $w_d$ per unit length, the rotation $\theta_1$ at supports due to sagging of the beam is evaluated from Fig. 4.22.

$$\theta_1 = \left(\frac{\frac{1}{2} \times \frac{2}{3} \times L\, w_d L^2/8}{EI}\right) = \left(\frac{w_d L^3}{24EI}\right)$$

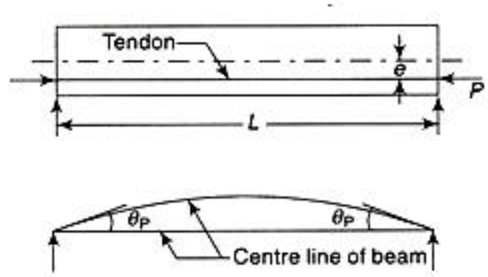


**Fig. 4.21** *Effect of prestressing force on rotation of concrete beam*

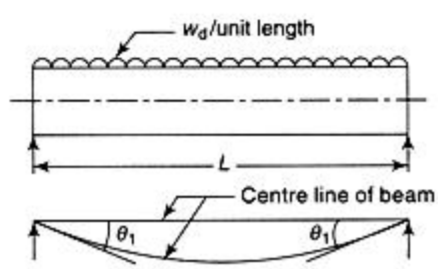


**Fig. 4.22** *Effect of transverse loads on rotation of concrete beam*

If the rotation due to loads is greater than that due to the prestressing force, the net rotation $\theta$ is given by,

$$\theta = (\theta_l - \theta_p)$$

Considering Fig. 4.23,

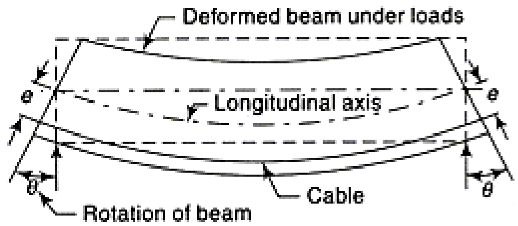


**Fig. 4.23** *Rotation of beam under the action of loads*

| | |
|---|---|
| Total elongation of the cable | $= 2e\theta$ |
| Strain in the cable | $= (2e\theta/L)$ |
| Increase in stress due to loading | $= \dfrac{(E_s 2e\theta)}{L}$ |

Generally, in the elastic range, any increase of loading on a prestressed member does not result in any significant change in the steel stress. In other words, the stress in steel is more or less constant in the elastic range of a prestressed member.

**3. A concrete beam of 10 m span, 100 mm wide and 300 mm deep is prestressed by 3 cables. The area of each cable is 200 mm$^2$ and the initial stress in the cable is 1200 N/mm$^2$. Cable 1 is parabolic with an eccentricity of 50 mm above the centroid at the supports and 50 mm below at the centre of span. Cable 2 is also parabolic with zero eccentricity at supports and 50 mm below the centroid at the centre of span. Cable 3 is straight with uniform eccentricity of 50 mm below the centroid. If the cables are tensioned from one end only. Estimate the percentage loss of stress in each cable due to friction. Assume μ = 0.35 and k = 0.0015 per m. (AUC Nov/Dec 2011)**

**Solution:**

$$y = (4e/L^2)x(L - x)$$

Slope at ends (at $x = 0$) $= dy/dx = (4e/L^2)(L - 2x) = (4e/L)$

***For cable 1***

$$\text{Slope at end} = \left(\frac{4 \times 10}{10 \times 100}\right) = 0.04$$

$\therefore$ Cumulative angle between tangents, $\alpha = (2 \times 0.04) = 0.08$ radians

***For cable 2***

$$\text{Slope at end} = \left(\frac{4 \times 5}{10 \times 100}\right) = 0.02$$

$\therefore$ Cumulative angle between tangents, $\alpha = (2 \times 0.02) = 0.04$ radians

Initial prestressing force in each cable, $P_0 = (200 \times 1200) = 24{,}0000$ N

If $P_x$ = prestressing force (stress) in the cable at the farther end,

$$P_x = P_o e^{-(\mu\alpha + kx)}$$

For small values of $(\mu\alpha + Kx)$, we can write

$$P_x = P_o[1 - (\mu\alpha + kx)]$$

$$\text{Loss of stress} = P_o(\mu\alpha + kx)$$

$$\text{cable 1} = P_o(0.35 \times 0.08 + 0.0015 \times 10) = 0.043\ P_o$$

$$\text{cable 2} = P_o(0.35 \times 0.04 + 0.0015 \times 10) = 0.029\ P_o$$

$$\text{cable 3} = P_o(0 + 0.0015 \times 10) = 0.015 P_o$$

If $P_o$ = Initial stress = 1200 N/mm$^2$

| Cable No. | Loss of stress, N/mm$^2$ | Percentage loss |
|---|---|---|
| 1 | 51.6 | 4.3 |
| 2 | 34.8 | 2.9 |
| 3 | 18.0 | 1.5 |

**4. A prestressed concrete beam of section 120 mm wide by 300 mm deep is used over an effective span of 6 m to support a uniformly distributed load of 4 kN/m, which includes the self weight of the beam. The beam is prestressed by a straight cable carrying a force of 180 kN and located at eccentricity of 50 mm. Determine the location of the thrust line in the beam and plot its position at quarter and central span section.**

**(AUC Apr/May 2012) (AUC May/June 2013, Nov/Dec 2013)**

**Solution:**

$$P = 180 \text{ kN}$$

$$E = 50 \text{ mm}$$

$$A = 36 \times 10^3 \text{ mm}^2$$

$$Z = 18 \times 10^5 \text{ mm}^3$$

Stresses due to prestressing force

$$P/A = \left(\frac{180 \times 10^3}{36 \times 10^3}\right) = +5 \text{ N/mm}^2$$

$$Pe/Z = \left(\frac{180 \times 10^3 \times 50}{18 \times 10^3}\right) = +5 \text{ N/mm}^2$$

Bending moment at the centre of the span $= (0.125 \times 4 \times 6^2)$

$$= 18 \text{ kN m}$$

$$\text{Bending stresses at top and bottom} = \left(\frac{18 \times 10^6}{18 \times 10^5}\right) = \pm 10 \text{ N/mm}^2$$

Resultant stresses at the central section:

$$\text{At top} = (5 - 5 + 10) = 10 \text{ N/mm}^2$$

$$\text{At bottom} = (5 + 5 - 10) = 0 \text{ N/mm}^2$$

$$\text{Shift of pressure line from cable line} = M/P = \left(\frac{18 \times 10^6}{18 \times 10^4}\right) = 100 \text{ mm}$$

$$\text{Bending moment at quarter span section} = (3/32)\, qL^2 = (3/32) \times 4 \times 6^2$$

$$= 13.5 \text{ kN m}$$

$$\text{Bending stress at top and bottom} = \left(\frac{13.5 \times 10^6}{18 \times 10^5}\right) = 7.5 \text{ N/mm}^2$$

Resultant stresses at the quarter span section:

$$\text{At top} = (5 - 5 + 7.5) = 7.5 \text{ N/mm}^2$$

$$\text{At bottom} = (5 + 5 - 7.5) = 2.5 \text{ N/mm}^2$$

$$\text{Shift of pressure-line from cable line } M/P = \left(\frac{13.5 \times 10^6}{18 \times 10^4}\right) = 75 \text{ mm}$$

The location of pressure line is shown in Fig. 4.13.

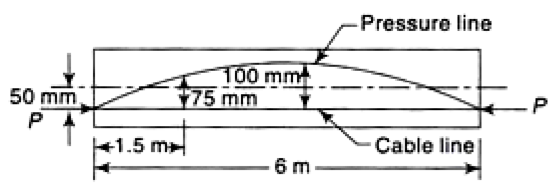


**Fig. 4.13** *Location of pressure line in the prestressed beam*

5. **A rectangular concrete beam 360 mm deep and 200 mm wide is prestressed by means of fifteen 5 mm diameter wires located 65 mm from the bottom of the beam and three 5 mm wires, located 25 mm from the top of the beam. If the wires are initially tensioned to a stress of 840 N/mm$^2$, calculate the percentage loss of stress due to elastic deformation of concrete only. $E_S$ = 210 kN/mm$^2$ and $E_C$ = 31.5 kN/mm$^2$. (AUC Nov/Dec 2013)**
   **Solution:**

$$E_s = 210 \text{ kN/mm}^2$$

$$E_c = 31.5 \text{ kN/mm}^2$$

Position of the centroid of the wires from the soffit of the beam,

$$y = \left[\frac{(15 \times 65) + (3 \times 275)}{(15 + 3)}\right] = 100 \text{ mm}$$

$$\therefore \quad \text{Eccentricity } e = (150 - 100) = 50 \text{ mm}$$

$$\text{Area of concrete } A = (200 \times 300) = 6 \times 10^4 \text{ mm}^2$$

$$\text{Second moment of area } I = \left(\frac{(200 \times 300^3)}{12}\right) = 45 \times 10^7 \text{ mm}^4$$

$$\text{Prestressing force } P = (840)(18 \times 19.7) = (3 \times 10^5) \text{ N}$$

$$= 300 \text{ kN}$$

Stresses in concrete:

$$\text{At the level of top wires} = \left(\frac{300 \times 10^3}{6 \times 10^4}\right) - \left(\frac{300 \times 10^3 \times 50 \times 125}{45 \times 10^7}\right)$$

$$= 0.83 \text{ N/mm}^2$$

$$\text{At the level of bottom wires} = \left(\frac{300 \times 10^3}{6 \times 10^4}\right) + \left(\frac{300 \times 10^3 \times 50 \times 85}{45 \times 10^7}\right)$$

$$= 7.85 \text{ N/mm}^2$$

$$\text{Modular ratio } (\alpha_e) = \left(\frac{210}{31.5}\right) = 6.68$$

$$\text{Loss of stress in wires at the top} = (6.68 \times 0.83) = 5.55 \text{ N/mm}^2$$

$$\text{Loss of stress in wires at the bottom} = (6.68 \times 7.85) = 52.5 \text{ N/mm}^2$$

Percentage loss of stress:

$$\text{For wires at top} = \left(\frac{5.55}{840} \times 100\right) = 0.66\%$$

$$\text{For wires at bottom} = \left(\frac{52.5}{840} \times 100\right) = 6.25\%$$

6. **A prestressed concrete beam, 200 mm wide and 300 mm deep is used over an effective span of 6 m to support an imposed load of 4 kN/m. the density of concrete is 24 kN/m$^3$. Find the magnitude of the eccentric prestressing force located at 100 mm from the bottom of the beam which would nullify the bottom fibre stress due to loading.**

**(AUC Apr/May 2011)**

**Solution:**

$$A = (200 \times 300) = 6 \times 10^4 \text{ mm}^2$$

$$Z_b = Z_t = \left(\frac{200 \times 300^2}{6}\right)$$

$$= 3 \times 10^6 \text{ mm}^3$$

$$g = (0.2 \times 0.3 \times 24) = 1.44 \text{ kN/m}$$

$$M_g = (0.125 \times 1.44 \times 6^2) = 6.48 \text{ kN m}$$

$$M_q = (0.125 \times 4 \times 6^2) = 18 \text{ kN m}$$

Tensile stress at the bottom fibre due to dead and live loads

$$= \left[\frac{(6.48 + 18)10^6}{3 \times 10^6}\right] = 8.16 \text{ N/mm}^2$$

If $P$ = eccentric prestressing force ($e$ = 50 mm), for zero stress at the soffit of the beam under loads

$$(P/A) + (Pe/Z_b) = 8.16$$

$$\therefore \quad P\left(\frac{1}{6 \times 10^4} + \frac{50}{3 \times 10^6}\right) = 8.16$$

$$P = 244.8 \text{ kN}$$

The magnitudes of the computed prestressing forces clearly indicate the advantages of eccentric prestressing in flexural members subjected to transverse loads.

7. **A pretensioned beam 200 mm x 300 mm is prestressed by 10 wires each of 7 mm diameter, initially stressed to 1200 MPa with their centroids located 100 mm from the soffit. Estimate the final percentage loss of stress due to elastic deformation, creep, shrinkage and relaxation. Assume relaxation of steel stress = 60 MPa, $E_S$ = 210 GPa, $E_C$ = 36.9 GPa, creep coefficient = 1.6 and residual shrinkage strain = 3 x $10^{-4}$.**

**(AUC Apr/May 2011)**

**Solution:**

$$A_c = (6 \times 10^4) \text{ mm}^2$$

$$E_c = 36900 \text{ N/mm}^2$$

$$I = 45 \times 10^7 \text{ mm}^4$$

$$\alpha_e = (E_s/E_c) = 5.7$$

$$P = (1200)(10 \times 38.5) = (462 \times 10^3) \text{ N}$$

$$= 462 \text{ kN}$$

Stress in concrete at the level of steel is given by

$$f_c = \left[\frac{462 \times 10^3}{6 \times 10^4} + \frac{(462 \times 10^3 \times 50)50}{45 \times 10^7}\right]$$

$$= 10.3 \text{ N/mm}^2$$

Loss of stress due to elastic deformation of concrete

$$= (5.7 \times 10.3) = 58.8 \text{ N/mm}^2$$

Force in wires immediately after transfer = (1200 – 58.8) 38.5

$$= 440\,000 \text{ N} = 440 \text{ kN}$$

Stress in concrete at the level of steel is given by

$$f_c = \left[\frac{440 \times 10^3}{6 \times 10^4} + \frac{(440 \times 10^3 \times 50)50}{45 \times 10^7}\right] = 978 \text{ N/mm}^2$$

*losses of prestress*

1. Elastic deformation $= 58.8\ \text{N/mm}^2$
2. Creep of concrete $= (1.6 \times 9.78 \times 5.7)$ $= 89.2\ \text{N/mm}^2$
3. Shrinkage of concrete $= (3 \times 10^{-4})(210 \times 10^3)$ $= 63.0\ \text{N/mm}^2$
4. Relaxation of steel stress $= (5/100)\ 1200$ $= 60.0\ \text{N/mm}^2$

Total loss $= 271.0\ \text{N/mm}^2$

Final stress in wires $= (1200 - 271.0) = 929.0\ \text{N/mm}^2$

$$\text{Percentage loss} = \left(\frac{271.0}{1200} \times 100\right) = 22.58\%$$

8. **A concrete beam with a rectangular section, 120 mm wide and 300 mm deep, is stressed by a straight cable carrying an effective force of 200 kN. The span of the beam is 6 m. The cable is straight with a uniform eccentricity of 50 mm. if the beam has an uniformly distributed load of 6 kN/m. $E_C$ = 38 kN/mm$^2$. Estimate the deflection at the centre of span for the following cases:**
   i. **Prestress + self weight of the beam**
   ii. **Prestress + self weight + live load. (AUC Nov/Dec 2010, May/Jun 2013, Apr/May 2010)**

**Solution:**

$P = 200$ kN

$g = 0.86$ N/mm

$L = 6000$ mm

$I = (27 \times 10^7)$ mm$^4$

$e = 50$ mm

$E = 38$ kN/mm$^2$

Deflection due to the prestressing force is computed as,

$$a_p = \left[\frac{-PeL^2}{8EI}\right] = -\left[\frac{200 \times 50 \times 6000^2}{8 \times 38 \times 27 \times 10^7}\right]$$

$= -4.38$ mm (Upwards)

Deflection due to self-weight of the beam is

$$a_g = \left[\frac{5gL^4}{384EI}\right] = -\left[\frac{5 \times 0.86 \times 6000^4}{384 \times 38 \times 10^3 \times 27 \times 10^7}\right]$$

$= 1.40$ mm (downwards)

Deflection due to live load on the beam is

$$a_w = \left[\frac{5wL^4}{384EI}\right] = \left[\frac{5 \times 0.006 \times (6 \times 1000)^4}{384 \times 38 \times 27 \times 10^7}\right]$$

$= 9.87$ mm (downwards)

(a) Deflection due to (prestress + self-weight) = [1.40 – 4.38]

= –2.98 mm (upwards)

(b) Deflection due to (prestress + self-weight + live load ) = [1.40 – 4.38 + 9.87]

= 6.89 mm (downwards)

9. **A simply supported PSC beam of cross section 100 mm wide and 250 mm deep is loaded with a uniformly distributed load of magnitude 1.2 kN/m on a span of 8 m. Obtain the stress distribution at mid span by stress and strength concept, if the prestressing force is 250 kN applied eccentrically all along with its centre of gravity at 40 mm. Assuming the density of concrete as 24 kN/m$^3$. (AUC Nov/Dec 2012)**

**Solution:**

Prestressing force = $P$ = 250 kN

Cross-sectional area = $A = (100 \times 250) = (25 \times 10^3)$ mm$^2$

Eccentricity = $e$ = 40 mm

Self-weight of beam = $g = (0.1 \times 0.25 \times 24)$

= 0.6 kN/m

Live load on the beam = $q$ = 1.2 kN/m

$b$ = 100 mm, $d$ = 250 mm, Span = $L$ = 8 m

∴ Total load on the beam = $w = (g + q) = (0.6 + 1.2)$

= 1.8 kN/m

Section Modulus

$$Z = \left(\frac{bd^2}{6}\right) = \left(\frac{100 \times 250^2}{6}\right)$$

$$= (1.04 \times 10^6) \text{ mm}^3$$

Bending moment at the centre-of-span section

$$M = \left(\frac{wL^2}{8}\right) = \left(\frac{1.8 \times 8^2}{8}\right)$$

$$= 14.4 \text{ kN m}$$

Stress due to loads

$$\left(\frac{M}{Z}\right) = \left(\frac{14.4 \times 10^6}{1.04 \times 10^6}\right) = \pm 13.8 \text{ N/mm}^2$$

Prestress at top and bottom fibres

$$\left[\frac{P}{A} \pm \frac{Pe}{Z}\right] = \left[\frac{250 \times 10^3}{25 \times 10^3} \pm \frac{250 \times 10^3 \times 40}{1.04 \times 10^6}\right] = [10 \pm 9.6] \text{ N/mm}^2$$

Resultant stresses

At top fibre = (10 – 9.6 + 13.8) = 14.2 N/mm$^2$ (Compression)

At bottom fibre = (10 + 9.6 – 13.8) = 5.8 N/mm$^2$ (Compression)

**10. A straight pretensioned prestressed concrete member 12 m long with a cross section of 400 mm wide and 500 mm deep is concentrically post tensioned by four tendons of 250 mm$^2$ each. The tendons are stressed one after another to the stress of 1000 N/mm$^2$. The eccentricity of prestressing force is 100 mm at the centre of the span. Compute the loss of prestress due to elastic shortening of concrete. How can the loss be counteracted?**

**(AUC Nov/Dec 2012)**

**Solution:**

Area of steel, $A_S$ = 250 mm$^2$

Area of total prestressing steel, $A_P$ = 4 x 250 = 1000 mm$^2$

Area of concrete section, A = 400 x 500 = 200000 mm$^2$

Moment of inertia, I = $\frac{b\ X\ d^3}{12} = \frac{400\ X\ 500^3}{12}$ = 4.17 x 10$^9$ mm$^4$

Prestressing force, P = stress x area = 1000 x 1000 = 1000 kN

Eccentricity of prestressing force, e = 100 mm

Prestress at top fibre, $f_t$ = $\frac{P}{A} + \frac{Pe}{I}y = \frac{1000\ X\ 10^3}{200000} - \frac{1000\ X\ 10^3\ X\ 100}{4.17\ X\ 10^9}\ X\ 250$

= - 1 N/mm$^2$

Prestress at bottom fibre, $f_b$ = $\frac{P}{A} - \frac{Pe}{I}y = \frac{1000\ X\ 10^3}{200000} + \frac{1000\ X\ 10^3\ X\ 100}{4.17\ X\ 10^9}\ X\ 250$

= 11 N/mm$^2$

Assume, Modular ratio, $\alpha_e$ = 6

Loss of prestress in top fibre = m $f_t$ A = 6 x 1 x 1000 = 6000 N

Loss of prestress in bottom fibre = m $f_b$ A = 6 x 11 x 1000 = 66000 N

Total loss of prestress = 72000 N

Percentage loss = $\frac{72}{1000}\ X\ 100 = 7.2\ \%$

**11. Explain the factors influencing deflections? (AUC Apr/May 2012)**

The deflections of prestressed concrete members are influenced by the following salient factors:

1. Imposed load and self-weight
2. Magnitude of the prestressing force
3. Cable profile
4. Second moment of area of cross section
5. Modulus of elasticity of concrete
6. Shrinkage, creep and relaxation of steel stress
7. Span of the member
8. Fixity conditions

In the precracking stage, the whole cross section is effective and the deflections in this stage are computed by using the second moment of area of the gross concrete section. The computation of short-term or instantaneous deflections, which occur immediately after transfer of prestress and on application of loads, is conveniently done by using Mohr's theorems.

In the post-cracking stage, a prestressed concrete beam behaves in a manner similar to that of a reinforced concrete beam and the computation of deflections in this stage is made by considering moment–curvature relationships which involve the section, properties of the cracked beam.

In both cases, the effect of creep and shrinkage of concrete is to increase the long-term deflections under sustained loads, which is estimated by using empirical methods that involve the use of effective (long term) modulus of elasticity or by multiplying short-term deflections by suitable factors.

**12. What is the necessity of using supplementary or untensioned reinforcement in prestressed concrete members? (AUC Apr/May 2012)**

Reinforcement in prestressed members not tensioned with respect to the surrounding concrete before the application of loads. These are generally used in partially prestressed members. The untensioned reinforcement is required in the cross section of a prestressed member for various reasons such as to resist the differential shrinkage, temperature effects and handling stresses. Hence this reinforcement can cater for the serviceability requirements such as control of cracking and partially for the ultimate limit state of collapse which can result in considerable reductions in the costlier high tensile steel. The saving in prestressing steel contributes to an overall saving in cost of the structure.

Fully prestressed members are prone to excessive upward deflections especially in bridge structures where dead loads form a major portion of the total service loads and these deflections may increase with time due to the effect of creep. It is also well established that fully prestressed members due to their higher rigidity have a lower energy absorption capacity in comparison with partially prestressed members which exhibit a ductile behaviour. There is no disadvantage in allowing tensile stress of about 5 $N/mm^2$ in a concrete where the strains are appropriately guided. High tensile steel and mild steel have been used as untensioned reinforcement. The present practice is to use high yield strength deformed bars which are considerably cheaper than prestressing steel and at the same time have higher yield strength and better crack control.

**13. A prestressed concrete beam of rectangular section 375 mm wide and 750 mm deep has a span of 12.5 m. the effective prestressing force is 1520 kN at an eccentricity of 150 mm. the dead load of the beam is 7 kN/m and the beam has to carry a live load of 12.5 kN/m. determine the extreme stresses in concrete**

**i. At the end section.**

**ii. At the mid section without the action of the live load.**

**iii. At the mid section with the action of the live load. (AUC Nov/Dec 2010)**

**Solution:**

Prestressing force = P = 1520 kN

Area of cross section = A = 375 x 750 = 281.25 x $10^3$ $mm^2$

Eccentricity = e = 150 mm

Dead load on the beam = g = 7 kN/m

Live load on the beam = q = 12.5 kN/m

b = 375 mm, d = 750 mm, L = 12.5 m

Total load on the beam = w = ( g + q ) = ( 7 + 12.5 ) = 19.5 kN/m

Section modulus, Z = $\frac{b\ X\ d^2}{6} = \frac{375\ X\ 750^2}{6} = 35.16\ X\ 10^6\ mm^3$

Bending moment at the centre of span section,

$$M = \left(\frac{wl^2}{8}\right) = \left(\frac{19.5\ X\ 12.5^2}{8}\right) = 380.86\ kNm$$

Stress due to dead load**,**

$$\left(\frac{M}{Z}\right) = \left(\frac{7\ X\ 10^6}{35.16\ X\ 10^6}\right) = 0.20\ N/mm^2$$

Stress due to live load**,**

$$\left(\frac{M}{Z}\right) = \left(\frac{12.5\ X\ 10^6}{35.16\ X\ 10^6}\right) = 0.36\ N/mm^2$$

**i) At the end section:**

Stress at top = $\frac{P}{A} = \frac{1520\ X\ 10^3}{281.25\ X\ 10^3} = 5.4\ N/mm^2\ (Compression)$

Stress at bottom = $\frac{P}{A} = \frac{1520\ X\ 10^3}{281.25\ X\ 10^3} = 5.4\ N/mm^2\ (Compression)$

**ii) At the mid section without the action of the live load:**

Stress at top = $\frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z} = \frac{1520\ X\ 10^3}{281.25\ X\ 10^3} - \frac{1520\ X\ 10^3\ X\ 40}{35.16\ X\ 10^6} + 0.20$

$= 3.47\ N/mm^2\ (Compression)$

Stress at bottom = $\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = \frac{1520\ X\ 10^3}{281.25\ X\ 10^3} + \frac{1520\ X\ 10^3\ X\ 40}{35.16\ X\ 10^6} - 0.20$

$$= 6.93 \ N/mm^2 \ (Compression)$$

**iii) At the mid section with the action of the live load:**

Stress at top = $\frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z} = \frac{1520 \ X \ 10^3}{281.25 \ X \ 10^3} - \frac{1520 \ X \ 10^3 \ X \ 40}{35.16 \ X \ 10^6} + 0.56$

$$= 4.23 \ N/mm^2 \ (Compression)$$

Stress at bottom = $\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = \frac{1520 \ X \ 10^3}{281.25 \ X \ 10^3} + \frac{1520 \ X \ 10^3 \ X \ 40}{35.16 \ X \ 10^6} - 0.56$

$$= 6.57 \ N/mm^2 \ (Compression)$$