EC6303  SIGNALS AND SYSTEMS  L T P C 3 1 0 4

OBJECTIVES:
- To understand the basic properties of signal & systems and the various methods of classification
- To learn Laplace Transform & Fourier transform and their properties
- To know Z transform & DTFT and their properties
- To characterize LTI systems in the Time domain and various Transform domains

UNIT I CLASSIFICATION OF SIGNALS AND SYSTEMS 9
Continuous time signals (CT signals) - Discrete time signals (DT signals) - Step, Ramp, Pulse, Impulse, Sinusoidal, Exponential, Classification of CT and DT signals - Periodic & Aperiodic signals, Deterministic & Random signals, Energy & Power signals - CT systems and DT systems- Classification of systems – Static & Dynamic, Linear & Nonlinear, Time-variant & Time-invariant, Causal & Noncausal, Stable & Unstable.

UNIT II ANALYSIS OF CONTINUOUS TIME SIGNALS 9
Fourier series analysis- spectrum of Continuous Time (CT) signals- Fourier and Laplace Transforms in CT Signal Analysis - Properties.

UNIT III LINEAR TIME INVARIANT- CONTINUOUS TIME SYSTEMS 9
Differential Equation-Block diagram representation-impulse response, convolution integrals-Fourier and Laplace transforms in Analysis of CT systems

UNIT IV ANALYSIS OF DISCRETE TIME SIGNALS 9
Baseband Sampling - DTFT – Properties of DTFT - Z Transform – Properties of Z Transform

UNIT V LINEAR TIME INVARIANT-DISCRETE TIME SYSTEMS 9
Difference Equations-Block diagram representation-Impulse response - Convolution sum- Discrete Fourier and Z Transform Analysis of Recursive & Non-Recursive systems

TOTAL (L:45+T:15):        60 PERIODS

OUTCOMES: Upon the completion of the course, students will be able to:
- Analyze the properties of signals & systems
- Apply Laplace transform, Fourier transform, Z transform and DTFT in signal analysis
- Analyze continuous time LTI systems using Fourier and Laplace Transforms
- Analyze discrete time LTI systems using Z transform and DTFT

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UNIT – I
CLASSIFICATION OF SIGNALS AND SYSTEMS

1.1 INTRODUCTION:
A signal, as stated before is a function of one or more independent variables. A signal is a quantitative description of a physical phenomenon, event or process. More precisely, a signal is a function, usually of one variable in time. However, in general, signals can be functions of more than one variable, e.g., image signals. Signals are functions of one or more variables.

Systems respond to an input signal by producing an output signal. Examples of signals include:

1. A voltage signal: voltage across two points varying as a function of time.
2. A force pattern: force varying as a function of 2-dimensional space.
3. A photograph: color and intensity as a function of 2-dimensional space.
4. A video signal: color and intensity as a function of 2-dimensional space and time.

A continuous-time signal is a quantity of interest that depends on an independent variable, where we usually think of the independent variable as time. Two examples are the voltage at a particular node in an electrical circuit and the room temperature at a particular spot, both as functions of time.

A discrete-time signal is a sequence of values of interest, where the integer index can be thought of as a time index, and the values in the sequence represent some physical quantity of interest.

A signal was defined as a mapping from a set of the independent variable (domain) to the set of the dependent variable (co-domain). A system is also a mapping, but across signals, or across mappings. That is, the domain set and the co-domain set for a system are both sets of signals, and corresponding to each signal in the domain set, there exists a unique signal in the co-domain set.

System description

The system description specifies the transformation of the input signal to the output signal. In certain cases, a system has a closed form description. E.g. the continuous-time system with description \( y(t) = x(t) + x(t-1) \); where \( x(t) \) is the input signal and \( y(t) \) is the output signal.

1.2 Continuous-time and discrete-time systems

- Physically, a system is an interconnection of components, devices, etc., such as a computer or an aircraft or a power plant.
- Conceptually, a system can be viewed as a black box which takes in an input signal \( x(t) \) (or \( x[n] \)) and as a result generates an output signal \( y(t) \) (or \( y[n] \)).
- A system is continuous-time (discrete-time) when its I/O signals are continuous-time (discrete-time).
1.3 Elementary Signals:
The elementary signals are used for analysis of systems. Such signals are,
- Step
- Impulse
- Ramp
- Exponential
- Sinusoidal

1.3.1 Unit step signal:
- Unit Step Sequence: The unit step signal has amplitude of 1 for positive value and amplitude of 0 for negative value of independent variable.
- It have two different parameter such as CT unit step signal \( u(t) \) and DT unit step signal \( u(n) \).
- The mathematical representation of CT unit step signal \( u(t) \) is given by,

1.3.2 Ramp Signal:
- The amplitude of every sample is linearly increased with the positive value of independent variable.
- Mathematical representation of CT unit ramp signal is given by,

The Ramp function

\[
r(t) = t \cdot u(t)
\]

1.3.3 Unit impulse function:
- Amplitude of unit impulse approaches 1 as the width approaches zero and it has zero value at all other values.
- The mathematical representation of unit impulse signal for CT is given by,

\[
\delta(t) = \begin{cases} 
\infty & \text{if } t = 0 \\
0, & \text{otherwise}
\end{cases}
\]

\[
\int_{-\infty}^{\infty} \delta(t) dt = \begin{cases} 
1, & t > 0 \\
0, & t < 0
\end{cases}
\]

- It is used to determine the impulse response of system.

1.3.4 Sinusoidal signal:
- A continuous time sinusoidal signal is given by,

\[
x(t) = A \cos \left( \Omega_0 t + \alpha \right)
\]

Where, \( A \) – amplitude \( \alpha \) – phase angle in radians
1.3.4 **Exponential signal:**
- It is exponentially growing or decaying signal.
- Mathematical representation for CT exponential signal is,

\[ x(t) = Ce^{at}, \quad \text{where } C, a \in \mathbb{C} \]

1.4 **Classification of CT and DT signals:**

- **Periodic and non-periodic Signals**
  A periodic function is one which has been repeating an exact pattern for an infinite period of time and will continue to repeat that exact pattern for an infinite time. That is, a periodic function \( x(t) \) is one for which

\[ x(t) = x(t+nT) \]

for any integer value of \( n \), where \( T > 0 \) is the period of the function and \(-\infty < t < \infty\). The signal repeats itself every \( T \) sec. Of course, it also repeats every \( 2T, 3T \) and \( nT \). Therefore, \( 2T, 3T \) and \( nT \) are all periods of the function because the function repeats over any of those intervals. The minimum positive interval over which a function repeats itself is called the fundamental period \( T_0 \). The fundamental frequency \( f_0 \) of a periodic function is the reciprocal of the fundamental period \( f_0 = 1/T_0 \). It is measured in Hertz and is the number of cycles (periods) per second. The fundamental angular frequency \( \omega_0 \) measured in radians per second is \( \omega_0 = 2\pi f_0 = 2\pi f_0 \). A signal that does not satisfy the condition in (2.1) is said to be a periodic or non-periodic.

- **Deterministic and Random Signals**
  Deterministic Signals are signals who are completely defined for any instant of time, there is no uncertainty with respect to their value at any point of time. They can also be described mathematically, at least approximately. Let a function be defined as

\[ \text{tri}(t) = \begin{cases} 1 - |t|, & -1 < t < 1 \\ 0, & \text{otherwise} \end{cases} \]
A random signal is one whose values cannot be predicted exactly and cannot be described by any exact mathematical function, they can be approximately described.

- **Energy and Power Signals:**
  Consider $v(t)$ to be the voltage across a resistor $R$ producing a current $i(t)$. The instantaneous power $p(t)$ per ohm is defined as,

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t)$$

For an arbitrary continuous-time signal $x(t)$, the normalized energy content $E$ of $x(t)$ is defined as,

$$E = \int_{-\infty}^{\infty} i^2(t) \, dt \text{ joules}$$

The normalized average power $P$ of $x(t)$ is defined as,

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} i^2(t) \, dt \text{ watts}$$

Similarly, for a discrete-time signal $x[n]$, the normalized energy content $E$ of $x[n]$ is defined as,

$$E = \sum_{n = -\infty}^{\infty} |x[n]|^2$$

The normalized average power $P$ of $x[n]$ is defined as,

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$
1.5 **CT Systems and DT Systems:**
A system is defined as a physical device which contains set of elements or functional blocks and that generates a response or output signal for a given input.

1.6 **Classification of systems:**
The systems are classified as,
- Static & dynamic system
- Time invariant and variant system
- Linear and non linear system
- Causal and non causal system
- Stable and unstable system

1.6.1 **Static and dynamic system:**
- Static system is said to be a memoryless system.
- The output does not depend the past or future input.
- It only depends the present input for an output.
  
  \[ y(n) = x(n) \]  
  
- Dynamic system is said to be as system with memory.
- Its output depend the past values of input for an output.
  
  \[ Y(n) = x(n) + x(n - 1) \]  
- This static and dynamic systems are otherwise called as memoryless and system with memory.

1.6.2 **Systems with and without memory:**
- A system is called memory less if the output at any time \( t \) (or \( n \)) depends only on the input at time \( t \) (or \( n \)); in other words, independent of the input at times before of after \( t \) (or \( n \)). Examples of memory less systems:
  
  \[ y(t) = R \cdot x(t) \quad \text{or} \quad y[n] = (2x[n] - x^2[n])^2 \]

Examples of systems with memory:

\[ y(t) = \frac{1}{C} \int_{-\infty}^{t} x(\tau) d\tau \quad \text{or} \quad y[n] = x[n - 1]. \]

1.6.3 **Time invariant and time variant system:**
- If the time shifts in the input signals results in corresponding time shift in the output, then the system is called as time invariant.
- The input and output characteristics do not change with time.
- For a continuous time system,
  
  \[ f[x(t1 - t2)] = y(t1 - t2) \]
- For a discrete time system,
  
  \[ F[x(n - k)] = y(n - k) \]
• If the above relation does not satisfy, then the system is said to be a time variant system.
• A system is called time-invariant if the way it responds to inputs does not change over time:
  \[ x(t) \rightarrow y(t) \quad \Rightarrow \quad x(t - t_0) \rightarrow y(t - t_0), \quad \text{for any} \ t_0 \]
  \[ x[n] \rightarrow y[n] \quad \Rightarrow \quad x[n - n_0] \rightarrow y[n - n_0], \quad \text{for any} \ n_0. \]

Examples of time-invariant systems:
• The RC circuit considered earlier provided the values of R or C are constant.
  \[ y[n] = x[n - 1]. \]

Examples of time-varying systems:
• The RC circuit considered earlier if the values of R or C change over time.
  \[ y(t) = x(2t) \quad \text{since} \]
  \[ x(t) \rightarrow x(2t) \quad \text{but} \quad x(t - t_0) \rightarrow x(2t - t_0). \]
• Most physical systems are slowly time-varying due to aging, etc. Hence, they can be considered time-invariant for certain time periods in which its behavior does not change significantly.

1.6.4 Linear and non linear system:
• A system is said to be linear if it satisfies the superposition principle.
• Superposition principle states that the response to a weighted sum of input signal be equal to the weighted sum of the output corresponding to each of the individual input signal.
• The continuous system is linear if,
  \[ F[a_1x_1(t) + a_2x_2(t)] = a_1y_1(t) + a_2y_2(t) \]
• The discrete system is linear if,
  \[ F[a_1x_1(n) + a_2x_2(n)] = a_1y_1(n) + a_2y_2(n) \]
• Otherwise the system is non linear.
• A system is called linear if its I/O behavior satisfies the additivity and homogeneity properties:
  \[ x_1(t) \rightarrow y_1(t) \quad \left\{ \begin{array}{c}
  (x_1(t) + x_2(t)) \rightarrow (y_1(t) + y_2(t)) \\
  (ax_1(t)) \rightarrow (ay_1(t)) 
\end{array} \right. \]
  for any complex constant a.
• Equivalently, a system is called linear if its I/O behavior satisfies the superposition property:
where any complex constants a and b.

\[ \begin{align*}
  x_1(t) &\rightarrow y_1(t) \\
  x_2(t) &\rightarrow y_2(t)
\end{align*} \Rightarrow \begin{align*}
  (a x_1(t) + b x_2(t)) &\rightarrow (a y_1(t) + b y_2(t))
\end{align*} \]

1.6.5 **Causal and non causal system:**
- A causal system is one whose output depends upon the present and past input values.
- If the system depends the future input values, the system is said to be non causal.

Eg. for causal system,
\[ Y(t) = x(t) + x(t - 1) \]
\[ Y(n) = x(n) + x(n - 3) \]

Eg. For non causal system,
\[ Y(t) = x(t+3) + x^2(t) \]
\[ Y(n) = x(2n) \]

- A system is called causal or non-anticipative if the output at any time t (or n) depends only on the input at times t or before t (or n or before n); in other words, independent of the input at times after t (or n). All memory less systems are causal. Physical systems where the time is the independent variable are causal.
- Non-causal systems may arise in applications where the independent variable is not the time such as in the image processing applications.

**Examples of causal systems:**
\[ y(t) = \frac{1}{C} \int_{-\infty}^{t} x(\tau) d\tau \quad \text{or} \quad y[n] = x[n - 1]. \]

**Examples of non-causal systems:**
\[ y(t) = x(-t) \quad \text{or} \quad y[n] = \frac{1}{3} (x[n - 1] + x[n] + x[n + 1]) \]

1.6.6 **Stable and unstable system:**
- When every bounded input produces bounded output then the system is called as stable system or bounded input bounded output (BIBO stable).
- Otherwise the system is unstable.
- A system is called stable if it produces bounded outputs for all bounded inputs.
- Stability in a physical system generally results from the presence of mechanisms that dissipate energy, such as the resistors in a circuit, friction in a mechanical system, etc.
Sample Problems:
1. Determine whether the following systems are: i) Memoryless, ii) Stable, iii) Causal, iv) Linear, and v) Time-invariant.

i) \( y(n) = nx(n) \)
ii) \( y(t) = e^{x(t)} \)

Solution:

(i) \( y[n] = n x[n] \)
A. Output depends only on the present value of the input \( \Rightarrow \) Memoryless
B. \( |x[n]| \leq M < \infty \) Bounded input
   \( |y[n]| = n M |x[n]| \) as \( n \to \infty \) output is unbounded \( \Rightarrow \) Unstable
C. System does not depend on future values \( \Rightarrow \) Causal
D. System satisfies both superposition and homogeneity conditions. \( H \{ a x[n] + b y[n] \} = a y[t] + b y[n] \) \( \Rightarrow \) Linear
E. \( y[n - n_0] = (n - n_0) x[n - n_0] \neq H \{ x[n - n_0] \} \) \( \Rightarrow \) Time-variant

(ii) \( y(t) = e^{x(t)} \)
A. Memoryless, B. Stable, C. Causal

2. Determine whether the following systems are time invariant or not.

i) \( Y(t) = tx(t) \)
ii) \( Y(n) = x(2n) \)

Solution:

i) \( Y(t) = tx(t) \)
   \( Y(t) = T[x(t)] = tx(t) \)
   The output due to delayed input is,
   \( Y(t, T) = T[x(t - T)] = tx(t - t) \)
   If the output is delayed by \( T \), we get
   \( Y(t - T) = (t - T) x( t - T) \)
   The system does not satisfy the condition, \( y(t, T) = y(t - T) \).
   Then the system is time invariant.
ii) \[ Y(n) = x(2n) \]
\[ Y(n) = x(2n) \]
\[ Y(n) = T[x(n)] = x(2n) \]

If the input is delayed by \( K \) units of time then the output is,
\[ Y(n,k) = T[x(n-k)] = x(2n-k) \]

The output delayed by \( k \) units of time is,
\[ Y(n-k) = x[2(n-k)] \]

Therefore, \( y(n,k) \) is not equal to \( y(n-k) \). Then the system is time variant.
2 mark question and answer

1. Define unit impulse and unit step signals. [May 2010]

**Unit Impulse signal:**
Amplitude of unit impulse is 1 as its width approaches zero. Then it has zero value at all other values.

**Unit Step Signal:**
The unit step signal has amplitude of 1 for positive values of independent variable and amplitude of 0 for negative of independent variable.

2. Give the mathematical and graphical representation of CT (continuous time) and DT (discrete time) impulse function. [Dec 2013]

**CT and DT impulse function:**
Continuous time unit impulse is defined as
\[
\delta(t) = \begin{cases} 
1, & t = 0 \\
0, & t \neq 0 
\end{cases}
\]

Discrete time Unit impulse is defined as
\[
\delta[n] = \begin{cases} 
0, & n \neq 0 \\
1, & n = 0 
\end{cases}
\]

Unit impulse is also known as unit sample.

3. Define step and impulse function in discrete signals. [May 2012]

**Unit step sequence** is defined as
\[
u[n] = \begin{cases} 
1, & n \geq 0 \\
0, & n < 0 
\end{cases}
\]

The unit sample function is defined as
\[
\delta[n] = \begin{cases} 
1, & n = 0 \\
0, & n \neq 0 
\end{cases}
\]
4. State the two properties of unit impulse function. [Dec 2014]
   i) Shifting property:
   \[ \int_{-\infty}^{\infty} x(i) \delta(i) \, dt = x(0) \]
   ii) Replication property:
   \[ \int_{-\infty}^{\infty} x(\tau) \delta(i-\tau) \, dt = x(i) \]

5. Find the fundamental period of signal [Dec 2010]
   \[ x = \sin \left( \frac{7\pi}{3} + t \right) \]

   Solu:
   \[ x(t) = \sin \left( \frac{7\pi}{3} + t \right) \]
   \[ \text{time period } T = \frac{2\pi}{\omega} \]
   \[ \omega = \frac{7\pi}{3} \]
   \[ = \frac{6}{7} \text{ sec} \]

6. Check e time whether the discrete signal \( \sin 3n \) is periodic? [June 2013]
   - The frequency of the discrete time signal is 3, because it is not a multiple of \( \pi \).
   - Therefore the signal is aperiodic.

7. Distinguish between deterministic and random signals. [May 2011]
   (or)

   **Define random signal and deterministic signal.** [May 2013]

   **Random Signal:**
   It has some degree of uncertainty before it actually occurs. The random signal cannot be defined by mathematical expressions.

   **Deterministic Signal:**
   There is no uncertainty occurrence. It is completely represented by mathematical expressions.

8. Determine the period of the signal [Dec 2011]
   \[ x = 2cos(n\pi/4) \]

   Solu:
   \[ 2\pi f n = n\pi/4 \]
   \[ f = \frac{\pi}{4} \times \frac{1}{2\pi} = 1/8 \]

   We know that, \( f=1/T \)
   So \( T = 8\text{sec} \)
9. When is a system said to be memory less? Give an example. [May 2010]

If the system output does not depend the previous input, it only depends the present input. Then the system is called memory less or static system.

Eg:
\[ y(t) = 2x(t) + x(t) \]
\[ y(n) = x(n) + \sqrt{x(n)} \]

10. Define energy and power signals. [Dec 2010]

Energy Signal:
- A signal is said to be an energy signal if its normalized energy is non zero and finite.
- For an energy signal, \( P = 0 \).

i.e., \( 0 < E < \infty \)

Power Signal:
- A signal is said to be the power signal if it satisfies \( 0 < P < \infty \)
- For a power signal, \( E = \infty \)

11. What is the classification of system? [Dec 2009]

The classification of systems is,
(i). Linear and Non-Linear systems
(ii). Time invariant and Time varying systems.
(iii). Causal and Non causal systems.
(iv). Stable and unstable systems.
(v). Static and dynamic systems.
(vi). Invertible and non invertible systems.

12. Verify whether the system described by the equation \( y(t) = x(t)^2 \) is linear and time invariant.

- The system is linear since output is direct function of input.
- The system is time variant since time parameter is squared in the given equation.
13. Draw the signal \( x(n) = u(n) - u(n-3) \)  

\[
\begin{array}{c}
\text{\( u(n) \)} \\
\text{0 1 2 3 4 5 6 7} \\
\text{\( u(n-3) \)} \\
\text{0 1 2 3 4 5 6 7 8 9} \\
\text{\( x(n) = u(n) - u(n-3) \)} \\
\text{0 1 2} \\
\text{Thus \( x(n) = [1 1 1] \)}
\end{array}
\]

[may 2011]

14. Check whether the following system is static/dynamic and casual/non casual \( y(n) = x(2n) \).

- If \( n=1 \), \( y(1) = x(2) \). This means system requires memory. Hence it is dynamic system.
- Since \( y(1) = x(2) \), the present output depends upon future input. Hence the system is non casual.

15. Distinguish between static and dynamic system.

**Static system:**
- Does not require memory
- Impulse response is of the form \( h(t) = c \omega(t) \)

**Dynamic system:**
- Requires memory
- Impulse response can be any form except \( h(t) = c \omega(t) \)
Unit – II
Analysis of Continuous Time Signals

2. Fourier series analysis:

Fourier series: a complicated waveform analyzed into a number of harmonically related sine and cosine functions

A two parts tutorial on Fourier series. In the first part an example is used to show how Fourier coefficients are calculated and in a second part you may use an applet to further explore Fourier series of the same function.

Fourier series may be used to represent periodic functions as a linear combination of sine and cosine functions. If \( f(t) \) is a periodic function of period \( T \), then under certain conditions, its Fourier series is given by:

\[
f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right]
\]

where \( n = 1, 2, 3, \ldots \) and \( T \) is the period of function \( f(t) \). \( a_n \) and \( b_n \) are called Fourier coefficients and are given by

\[
a_0 = \frac{2}{T} \int_{0}^{T} f(t) \, dt
\]
\[
a_n = \frac{2}{T} \int_{0}^{T} f(t) \cos\left(\frac{2n\pi t}{T}\right) \, dt
\]
\[
b_n = \frac{2}{T} \int_{0}^{T} f(t) \sin\left(\frac{2n\pi t}{T}\right) \, dt
\]

2.1 Continuous Time Fourier Transform:

The Fourier expansion coefficient \( X[k] \) (in \( a_k \) OWN) of a periodic signal is \( x_T(t) = x_T(t + T) \) is

\[
X[k] = \frac{1}{T} \int_{T} x_T(t) e^{-j2\pi k \omega_0 t} \, dt \quad (k = 0, \pm 1, \pm 2, \cdots)
\]

and the Fourier expansion of the signal is:
which can also be written as:

\[ x_T(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} (TX[k])e^{j\omega_0 t} = \frac{\omega_0}{2\pi} \sum_{k=-\infty}^{\infty} X(k\omega_0)e^{j\omega_0 t} \]  

(a)

where \( X(k\omega_0) \) is defined as

\[ X(k\omega_0) \triangleq T X[k] = \int_T x_T(t)e^{-j\omega_0 t} dt \]  

(b)

When the period of \( x_T(t) \) approaches infinity \( T \to \infty \), the periodic signal \( x_T(t) \) becomes a non-periodic signal \( x(t) \) and the following will result:

Interval between two neighboring frequency components becomes zero:

\[ T \to \infty \implies \omega_0 = \frac{2\pi}{T} \to 0 \]

Discrete frequency becomes continuous frequency:

\[ k\omega_0 \big|_{\omega_0 \to 0} \implies \omega \]

Summation of the Fourier expansion in equation (a) becomes an integral:

\[ x(t) \triangleq \lim_{T \to \infty} x_T(t) = \lim_{\omega_0 \to 0} \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(k\omega_0)e^{j\omega_0 t}\omega_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \]

the second equal sign is due to the general fact:

\[ \lim_{\Delta x \to 0} \sum_{k=-\infty}^{\infty} f(k\Delta x)\Delta x = \int_{-\infty}^{\infty} f(x)dx \]
Time integral over in equation (b) becomes over the entire time axis:

\[ X(\omega) \triangleq \lim_{T \to \infty} X(k\omega_0) = \lim_{T \to \infty} \int_T x_T(t)e^{-j\omega_0t}dt = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \]

In summary, when the signal is non-periodic \( x(t) = \lim_{T \to \infty} x_T(t) \), the Fourier expansion becomes Fourier transform. The forward transform (analysis) is:

\[ X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad \text{or} \quad X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \]

and the inverse transform (synthesis) is:

\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \]

Note that \( X(\omega) \) is denoted by \( X(j\omega) \) in OWN.

Comparing Fourier coefficient of a periodic signal \( x_T(t) \) with with Fourier spectrum of a non-periodic signal \( x(t) \)

\[ X[k] = \frac{1}{T} \int_T x_T(t)e^{-jkw_0t}dt, \quad X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \]

we see that the dimension of \( X(\omega) \) is different from that of \( X[k] \):

\[ [X(\omega)] = [X[k]][\omega] = \frac{[X[k]]}{[\omega]} \]

If \( |X[k]|^2 \) represents the energy contained in the kth frequency component of a periodic signal \( x_T(t) \), then \( |X(\omega)|^2 \) represents the energy density of a non-periodic signal.
distributed along the frequency axis. We can only speak of the energy contained in a particular frequency band $\omega_1 < \omega < \omega_2$.

### 2.2 Inverse Transforms

If we have the full sequence of Fourier coefficients for a periodic signal, we can reconstruct it by multiplying the complex sinusoids of frequency $\omega_0 k$ by the weights $X_k$ and summing:

$$x(n) = \sum_{k=0}^{P-1} X_k e^{j\omega_0 kn}$$

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\omega_0 t}$$

We can perform a similar reconstruction for aperiodic signals:

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

These are called the **inverse transforms**.

### 2.4 Parseval's theorem:

**Parseval's theorem** states that,

If $x(n) \xrightarrow{DFT} X(\Omega)$

Then energy of the signal is given as,

$$E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\Omega)|^2 d\Omega$$

**Proof:** We know that energy of the signal is given as,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$|x(n)|^2$ is also equal to $x(n) x^*(n)$. Hence energy becomes,

$$E = \sum_{n=-\infty}^{\infty} x(n) x^*(n)$$

From equation 4.2.2, we can write the inverse DIFT of $x^*(n)$ as,

$$x^*(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X^*(\Omega) e^{-j\Omega n} d\Omega$$
2.5 Laplace Transform

► Laplace transform is a generalization of the Fourier transform in the sense that it allows “complex frequency” whereas Fourier analysis can only handle “real frequency”. Like Fourier transform, Lapalce transform allows us to analyze a “linear circuit” problem, no matter how complicated the circuit is, in the frequency domain in stead of in he time domain.

► Mathematically, it produces the benefit of converting a set of differential equations into a corresponding set of algebraic equations, which are much easier to solve. Physically, it produces more insight of the circuit and allows us to know the bandwidth, phase, and transfer characteristics important for circuit analysis and design.

► Most importantly, Laplace transform lifts the limit of Fourier analysis to allow us to find both the steady-state and “transient” responses of a linear circuit. Using Fourier transform, one can only deal with the steady state behavior (i.e. circuit response under indefinite sinusoidal excitation).

► Using Laplace transform, one can find the response under any types of excitation (e.g. switching on and off at any given time(s), sinusoidal, impulse, square wave excitations, etc.

![Laplace Transform Formula](image)

<table>
<thead>
<tr>
<th>Property</th>
<th>( F(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(t) )</td>
<td>Definition</td>
</tr>
<tr>
<td>( f_1(t) + f_2(t) )</td>
<td>Linearity</td>
</tr>
<tr>
<td>( Kf(t) )</td>
<td>Linearity</td>
</tr>
<tr>
<td>( \frac{df(t)}{dt} )</td>
<td>Differentiation</td>
</tr>
<tr>
<td>( \frac{d^2f(t)}{dt^2} )</td>
<td>Differentiation</td>
</tr>
<tr>
<td>( \int_0^\infty f(t) , dt )</td>
<td>Integration</td>
</tr>
<tr>
<td>( tf(t) )</td>
<td>Complex differentiation</td>
</tr>
<tr>
<td>( e^{-at}f(t) )</td>
<td>Complex translation</td>
</tr>
<tr>
<td>( f(t - a) u(t - a) )</td>
<td>Real translation</td>
</tr>
</tbody>
</table>

Application of Laplace Transform to Circuit Analysis
2.6 Properties of ROC of Laplace Transform:

1. The ROC of \( X(s) \) consists of strips parallel to the \( j\omega \) axis in the \( s \)-plane.
2. The ROC does not contain any poles.
3. If \( x(t) \) is of finite duration and is absolutely integrable, then the ROC is the entire \( s \)-plane.
4. If \( x(t) \) is a right sided signal, that is \( x(t) = 0 \) for \( t < t_0 < \infty \) then the ROC is of the form \( \text{Re}(s) > \alpha_{\text{max}} \), where \( \alpha_{\text{max}} \) equals the maximum real part of any of the poles of \( X(s) \).
5. If \( x(t) \) is a left sided, that is \( x(t) = 0 \) for \( t > t_1 > -\infty \), then the ROC is of the form \( \text{Re}(s) < \alpha_{\text{min}} \), where \( \alpha_{\text{min}} \) equals the minimum real part of any of the poles of \( X(s) \).
6. If \( x(t) \) is a two sided signal, than the ROC is of the form \( \alpha_1 < \text{Re}(s) < \alpha_2 \).
(2 mark questions)

   (i). The function $x(t)$ should be single valued within the interval $T_0$
   (ii). The function $x(t)$ should have atmost a finite number of discontinuities
   in the interval $T_0$
   (iii). The function $x(t)$ should have finite number of maxima and minima
   in the interval $T_0$
   (iv). The function should have absolutely integrable.

2. State any two properties of continuous time Fourier transform. [May 2010, Dec 2009]

   Convolution (Time) Property:
   It states that,
   \[ x(t) * y(t) \overset{FT}{\leftrightarrow} X(j\Omega)Y(j\Omega) \]

   Modulation Property (or) Frequency Shifting:
   It states that,
   \[ x(t) e^{j\Omega_0 t} \overset{FT}{\leftrightarrow} X(j\Omega - j\Omega_0) \]

3. Find the laplace transform of the signal $x(t) = e^{-at}u(t)$ [May 2010]

   Given $x(t) = te^{-at}u(t)$
   We know that,
   \[ X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t} \, dt = \int_{0}^{\infty} e^{-at}u(t)e^{-j\Omega t} \, dt = \int_{0}^{\infty} e^{-at}e^{-j\Omega t} \, dt \]
   \[ = \int_{0}^{\infty} e^{-(a+j\Omega)t} \, dt = \frac{1}{a+j\Omega} \]
   \[ X(\Omega) = \frac{1}{a+j\Omega} \]


<table>
<thead>
<tr>
<th>S.NO</th>
<th>Fourier Series</th>
<th>Fourier Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fourier series is calculated for periodic signals.</td>
<td>Fourier Transform is calculated for non-periodic as well as periodic signals.</td>
</tr>
<tr>
<td>2</td>
<td>Expands the signals in time domain.</td>
<td>Represents the signal in frequency domain</td>
</tr>
<tr>
<td>3</td>
<td>Three types of Fourier series such as trigonometric, Polar and Complex Exponential</td>
<td>Fourier transform has no such types.</td>
</tr>
</tbody>
</table>
5. State initial and final value theorem of Laplace transform. [DEC 2009, MAY-11]

Initial value theorem: \( x(0) = \lim_{S\to\infty} SX(S) \)

Final value theorem: \( x(t) = \lim_{S\to0} SX(S) \)

6. Define the Fourier transform pair for continuous time signal. (Or) Give synthesis and analysis equations of CT Fourier Transform. [NOV/DEC 2012]

**Fourier Transform:**
\[
X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt
\]

**Inverse Fourier Transform:**
\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\omega t} \, d\Omega
\]

7. Find inverse Fourier transform of \( X(\omega) = 2\pi \delta(\omega) \). [MAY/JUNE 2010]

**Inverse Fourier Transform:**
\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\omega t} \, d\Omega
\]

**[Note: \( \omega = \Omega \)]**
\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi \delta(\Omega) e^{j\omega t} \, d\Omega
\]
\[
= 1. \quad \text{Since} \delta(\Omega) = \begin{cases} 1 & \text{for } \Omega = 0 \\ 0 & \text{for } \Omega \neq 0 \end{cases}
\]

8. State the time scaling property of Laplace Transform. [MAY/JUNE 2013]

It states that,
If \( L[x(t)] = X(S) \) then, \( L[x(at)] = \frac{1}{|a|} X\left(\frac{S}{a}\right) \)

9. What is the Fourier Transform of a DC signal of amplitude 1? [MAY/JUNE 2013]

\[
W.K.T X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt
\]

Here \( x(t) = 1 \) then, \( F[1] = 2\pi \delta(\Omega) \)

10. Define region of convergence of the Laplace Transform. [NOV/DEC 2012]

For a given signal the range of values of \( s \), for which the integral \( \int_{-\infty}^{\infty} |x(t)| \, dt \) converges is called the region of convergence.

i.e., \( \int_{-\infty}^{\infty} |x(t)| e^{\sigma t} \, dt < \infty \)
11. State the relationship between fourier transform and laplace transform. [May 2015]

- The laplace transform is given by,
  \[ X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt \]

- The fourier transform is given by,
  \[ X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \]

- The laplace transform is same as fourier transform when \( s = j\omega \)

12. State any two properties of ROC of laplace transform \( X(s) \) of a signal \( x(t) \). [Jun 2014]

Properties of ROC:
- No poles lie in ROC.
- ROC of the causal signal is right hand sided. It is of the form \( \text{Re}(s) > a \).
- ROC of the non-causal signal is left hand sided. It is of the form \( \text{Re}(s) < a \).
- The system is stable if its ROC includes \( j\omega \) axis of s-plane.

13. Determine fourier series coefficients for signal \( \cos \pi t \) [May 2012]

\[ \cos \pi t = \frac{e^{j\pi t} + e^{-j\pi t}}{2} \]

Fourier series is given as,
\[ x(t) = \sum_{k=-\infty}^{\infty} X(K)e^{jk\pi t} \]


- Fourier transform,
  Analysis equation =>
  \[ X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \]

- Inverse fourier transform,
  Synthesis equation =>
  \[ X(f) = 1/2\pi \int_{-\infty}^{\infty} x(\omega)e^{j\omega t} d\omega \]

15. Obtain the fourier transform of \( X(f) = e^{-at}u(t) \), \( a > 0 \)

\[ X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt \]
16. What is the condition to be satisfied for the existence of Fourier transform for CT periodic signals? [dec 2011]

The function $x(t)$ should be absolutely integrable for the existence of Fourier transform.

i.e., $\int_{-\infty}^{\infty} |x(t)| dt < \infty$
UNIT III

LINEAR TIME INVARIANT – CONTINUOUS TIME SYSTEMS

3.1 System:
A system is an operation that transforms input signal \( x \) into output signal \( y \).

3.2 LTI Systems
• Time Invariant
  \[ X(t) \square y(t) \square x(t-t_0) \square y(t-t_0) \]
• Linearity
  \[ a_1 x_1(t)+ a_2 x_2(t) \square a_1 y_1(t)+ a_2 y_2(t) \]
  \[ a_1 y_1(t)+ a_2 y_2(t) = T[a_1 x_1(t)+a_2 x_2(t)] \]
• Meet the description of many physical systems
• They can be modeled systematically
  – Non-LTI systems typically have no general mathematical procedure to obtain solution

Differential equation:
• This is a linear first order differential equation with constant coefficients (assuming \( a \) and \( b \) are constants)

\[
\frac{d}{dt}y(t) - ay(t) = bx(t)
\]

The general nth order linear DE with constant equations is

\[
a_0 y(t) + a_1 \frac{d}{dt}y(t) + \ldots + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} y(t) + a_n \frac{d^n}{dt^n} y(t) = \\
b_0 x(t) + b_1 \frac{d}{dt}x(t) + \ldots + b_{m-1} \frac{d^{m-1}}{dt^{m-1}} x(t) + b_m \frac{d^m}{dt^m} x(t)
\]

which we can write as:

\[
\sum_{k=0}^{n} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{m} b_k \frac{d^k}{dt^k} x(t).
\]
Linear constant-coefficient differential equations In RC circuit

To introduce some of the important ideas concerning systems specified by linear constant-coefficient differential equations, let us consider a first-order differential equations:

$$\frac{dv(t)}{dt} + \frac{1}{RC}v(t) = \frac{1}{RC}v_0(t)$$

3.3 Block diagram representations

Block diagram representations of first-order systems described by differential and difference equations

3.4 Impulse Response

This impulse response signal can be used to infer properties about the system’s structure (LHS of difference equation or unforced solution). The system impulse response, $h(t)$ completely characterises a linear, time invariant system.
3.5 **Properties of System Impulse Response**

**Stable**
A system is stable if the impulse response is absolutely summable

**Causal**
A system is causal if $h(t)=0$ when $t<0$

**Finite/infinite impulse response**
The system has a finite impulse response and hence no dynamics in $y(t)$ if there exists $T>0$, such that: $h(t)=0$ when $t>T$

**Linear**
\[ ad(t) \square ah(t) \]

**Time invariant**
\[ d(t-T) \square h(t-T) \]

3.6 **Convolution Integral**
• An approach (available tool or operation) to describe the input-output relationship for LTI Systems

\[ X(t)=\delta(t) \rightarrow \text{LTI System} \rightarrow y(t)=h(t) \]

• In a LTI system
  – $d(t) \square h(t)$
  – Remember $h(t)$ is $T[d(t)]$
  – Unit impulse function \( \square \) the impulse response
• It is possible to use $h(t)$ to solve for any input-output relationship
• Any input can be expressed using the unit impulse function

3.6.1 **Convolution Integral - Properties**

- **Commutative**
  \[ x(t) \ast h(t) = h(t) \ast x(t) \]

- **Associative**
  \[ x(t) \ast [h_1(t) + h_2(t)] = [x(t) \ast h_1(t)] + [x(t) \ast h_2(t)] \]

- **Distributive**
  \[ x(t) \ast [h_1(t) \ast h_2(t)] = [x(t) \ast h_1(t)] \ast h_2(t) \]

- Thus, using commutative property:
  \[ x(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \]
D.1.1 Commutativity Property

By making the change of variable, \( \lambda = t - \tau \), in one form of the definition of CT convolution,

\[
x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau
\]

it becomes

\[
x(t) * h(t) = -\int_{-\infty}^{\infty} x(t - \lambda) h(\lambda) d\lambda = \int_{-\infty}^{\infty} h(\lambda) x(t - \lambda) d\lambda = h(t) * x(t)
\]

proving that convolution is commutative.

D.1.2 Associativity Property

Associativity can be proven by considering the two operations

\[
[x(t) * y(t)] * z(t) \quad \text{and} \quad x(t) [* y(t) * z(t)].
\]

Using the definition of convolution

\[
x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau
\]

we get

\[
[x(t) * y(t)] * z(t) = \left[ \int_{-\infty}^{\infty} x(\tau) y(t - \tau) d\tau \right] * z(t)
\]

D.1.3 Distributivity Property

Convolution is also distributive,

\[
x(t) * \left[ h_1(t) + h_2(t) \right] = x(t) * h_1(t) + x(t) * h_2(t).
\]

\[
x(t) * \left[ h_1(t) + h_2(t) \right] = \int_{-\infty}^{\infty} x(\tau) \left[ h_1(t - \tau) + h_2(t - \tau) \right] d\tau
\]

\[
x(t) * \left[ h_1(t) + h_2(t) \right] = \int_{-\infty}^{\infty} x(\tau) h_1(t - \tau) d\tau + \int_{-\infty}^{\infty} x(\tau) h_2(t - \tau) d\tau
\]

\[
x(t) * \left[ h_1(t) + h_2(t) \right] = x(t) * h_1(t) + x(t) * h_2(t)
\]
3.7 Properties of Laplace Transform:

<table>
<thead>
<tr>
<th>Properties</th>
<th>Time Domain</th>
<th>Laplace Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linearity</td>
<td>( a_1x_1(t) + a_2x_2(t) + \ldots + a_nx_n(t) )</td>
<td>( a_1X_1(s) + a_2X_2(s) + \ldots + a_nX_n(s) )</td>
</tr>
<tr>
<td>2. Frequency Shifting</td>
<td>( e^{-\alpha t}x(t) )</td>
<td>( F(s + \alpha) )</td>
</tr>
<tr>
<td>3. Time Delay</td>
<td>( x(t - \alpha)u(t - \alpha) )</td>
<td>( e^{-\alpha s}X(s) )</td>
</tr>
<tr>
<td>4. Time Scaling</td>
<td>( x(\alpha t) )</td>
<td>( \frac{1}{\alpha} X\left(\frac{s}{\alpha}\right) )</td>
</tr>
<tr>
<td>5. Time Differentiation</td>
<td>( \frac{d}{dt} x(t) )</td>
<td>( sX(s) - x(0^-) )</td>
</tr>
<tr>
<td>6. Time Integration</td>
<td>( \int_{-\infty}^{\infty} x(\tau)d\tau )</td>
<td>( \frac{X(s)}{s} + \frac{1}{s} \int_{-\infty}^{\infty} x(\tau)d\tau )</td>
</tr>
<tr>
<td>7. Initial Value Theorem</td>
<td>( \lim_{t \to 0^-} x(t) )</td>
<td>( \lim_{s \to \infty} sX(s) = x(0^+) )</td>
</tr>
<tr>
<td>8. Final Value Theorem</td>
<td>( \lim_{t \to \infty} x(t) )</td>
<td>( \lim_{s \to 0} sX(s) = x(\infty) )</td>
</tr>
<tr>
<td>9. Time Convolution</td>
<td>( x(t) * y(t) )</td>
<td>( X(s)Y(s) )</td>
</tr>
</tbody>
</table>

1. **Linearity**

Assume \( x(t) = a_1x_1(t) + a_2x_2(t) \) (\( a_1 \) and \( a_2 \) are time independent)

\[ X_i(s) = L[x_i(t)], \quad X_j(s) = L[x_j(t)] \]

Then

\[ X(s) = L[x(t)] = a_1X_1(s) + a_2X_2(s) \]

**Proof:**

\[
L[a_1x_1(t) + a_2x_2(t)] = \int_{0}^{\infty} (a_1x_1(t) + a_2x_2(t))e^{-st}dt \\
= \int_{0}^{\infty} a_1x_1(t)e^{-st} + a_2x_2(t)e^{-st}dt \\
= a_1\int_{0}^{\infty} x_1(t)e^{-st}dt + a_2\int_{0}^{\infty} x_2(t)e^{-st}dt \\
= a_1X_1(s) + a_2X_2(s) \\
\]

2. **Complex Frequency shift (s-shift) Theorem**

Assume \( y(t) = x(t)e^{-\alpha} \)

\[ X(s) = L[x(t)], \quad Y(s) = L[y(t)] \]

Then

\[ Y(s) = X(s + \alpha) \]

3. **Time Delay Theorem**

Assume \( L[x(t)u(t)] = X(s) \)

Then

\[ L[x(t-t_0)u(t-t_0)] = e^{-s^2t_0}X(s) \quad (t_0 > 0) \]
4. **Scaling**

Assume \( X(s) = L[x(t)] \) then \( L[x(at)] = \frac{1}{a} \frac{X(s)}{a} \)

**Restriction:** \( a > 0 \)

\( x(at) \leftarrow a \) times fast (if \( a > 1 \)) or slow (if \( a < 1 \)) as \( x(t) \)

**Proof:**

\[
L[x(at)] = \int_{0}^{\infty} x(at)e^{-st}dt = \frac{1}{a} \int_{0}^{\infty} x(\tau)e^{-\frac{s}{a}\tau}d\tau, \quad \text{set} \ \tau = at
\]

\[
= \frac{1}{a} X\left(\frac{s}{a}\right)
\]

5. **Time Differentiation**

Assume \( X(s) = L[x(t)] \)

Then \( L\left(\frac{dx(t)}{dt}\right) = sX(s) - x(0^-) \)

**Proof:**

1. **Definition**

\[
L\left(\frac{dx(t)}{dt}\right) = \int_{0}^{\infty} \frac{dx(t)}{dt}e^{-st}dt = \int_{0}^{\infty} e^{-st}dx(t)
\]

2. **Integration by parts:**

\[
\int_{a}^{b} u(t)v'(t)dt = [u(t)v(t)]_{a}^{b} - \int_{a}^{b} u'(t)v(t)dt
\]

Make the following substitution:

\[
z(t) \rightarrow v(t)
\]

\[
e^{-st} \rightarrow u(t)
\]

\[
L\left(\frac{d}{dt}x(t)\right) = \int_{0}^{\infty} e^{-st}dx(t)
\]

\[
= \sigma^{st}x(t)|_{t=\infty}^{0} - \int_{0}^{\infty} x(t)e^{-st}dt
\]

\[
= 0 - x(0^-) - (-\sigma) \int_{0}^{\infty} x(t)e^{-st}dt
\]

\[
= sX(s) - x(0^-)
\]

---

*In the ROC of \( X(s) \), we must have \( \lim_{s \to 0} [e^{s}x(t)] = 0 \) otherwise the Laplace integral which is the area under \( e^{s}x(t) \) from \( 0 \) to \( t = \infty \), will become \( \infty \).*
3.8 State variables and Matrix representation

- State variables represent a way to describe ALL linear systems in terms of a common set of equations involving matrix algebra.
- Many familiar properties, such as stability, can be derived from this common representation. It forms the basis for the theoretical analysis of linear systems.
- State variables are used extensively in a wide range of engineering problems, particularly mechanical engineering, and are the foundation of control theory.
- The state variables often represent internal elements of the system such as voltages across capacitors and currents across inductors.
- They account for observable elements of the circuit, such as voltages, and also account for the initial conditions of the circuit, such as energy stored in capacitors. This is critical to computing the overall response of the system.
- Matrix transformations can be used to convert from one state variable representation to the other, so the initial choice of variables is not critical.
- Software tools such as MATLAB can be used to perform the matrix manipulations required.
- Let us define the state of the system by an \( N \)-element column vector, \( x(t) \):

\[
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}
\]

Note that in this development, \( v(t) \) will be the input, \( y(t) \) will be the output, and \( x(t) \) is used for the state variables.
- Any system can be modeled by the following state equations:
- This system model can handle single input/single output systems, or multiple inputs and outputs.
- The equations above can be implemented using the signal flow graph shown to the below

![Signal Flow Graph](image)

- Consider the CT differential equations:

\[
\dot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_2 v(t)
\]

- A second-order differential equation requires two state variables:

\[
x_1(t) = y(t) \quad \text{and} \quad x_2(t) = \dot{y}(t)
\]

- We can reformulate the differential equation as a set of three equations:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= -a_2 x_1(t) - a_1 x_2(t) + b_2 v(t) \\
&\vdots \\
\dot{y}(t) &= x_1(t)
\end{align*}
\]
• We can write these in matrix form as:

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-a_0 & -a_1
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
b_0
\end{bmatrix} v(t)
\]

• This can be extended to an Nth order differential equation of this type:

\[
y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = b_0 v(t)
\]

The state variables are defined as \(x_i(t) = y^{(i-1)}(t), \ i = 1, 2, ..., N\)

The resulting state equation is

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
& \quad \vdots \\
\dot{x}_{N-1}(t) &= x_N(t) \\
\dot{x}_N(t) &= -\sum_{i=0}^{N-1} a_i x_{i+1}(t) + b_0 v(t)
\end{align*}
\]

Matrix representation

\[
A = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
-a_0 & -a_1 & -a_2 & \cdots & -a_{N-1}
\end{bmatrix}, \quad \quad B = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
b_0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0
\end{bmatrix}, \quad \quad D = 0
\]
(2 mark questions)

1. What is the overall impulse response \( h(t) \) when two systems with impulse response \( h_1(t) \) and \( h_2(t) \) are in parallel and in series? [MAY-10].
   (or)
   State the properties needed for interconnecting LTI systems. [MAY-09]
   For parallel connection, \( h(t) = h_1(t) + h_2(t) \)
   For series connection, \( h(t) = h_1(t) * h_2(t) \).

2. Write convolution integral of \( x(t) \) (or) Define convolution integral of continuous time systems. [DEC-10, MAY-10, MAY-11]
   The convolution integral is given as, \( y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau \).

3. Check whether the causal system with transfer function \( H(s) = 1/(s-2) \) is stable [DEC 2013]
   Here the pole lies at \( s = 2 \). Since the pole of causal system does not lie on the left side of \( j\omega \) axis, the system is not stable.

4. The impulse response of the LTI – CT system is given as \( h(t) = e^{-t}u(t) \). Determine transfer function and check whether the system is causal and stable.
   \( h(t) = e^{-t}u(t) \)
   Taking laplace transform, 
   \( H(s) = 1/(s+1) \)
   Here the pole lies at \( s = -1 \), i.e. located in left half of s-plane. Hence this system is causal and stable.

5. What are the conditions for a system to be LTI system? [DEC 2013]
   Input and output of an LTI system are related by,
   \( y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau \) i.e. convolution

6. What is the impulse response of two LTI systems connected in parallel? [MAY 2010]
   If the system are connected in parallel, having responses \( h_1(t) \) and \( h_2(t) \), then their overall response is given as,
   \( h(t) = h_1(t) + h_2(t) \)

7. Write \( N^{th} \) order differential equation. [DEC-10]
   The \( N^{th} \) order differential equation can be written as,
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\[ \sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \]

Here \( N \geq M \).

8. What is the condition for LTI system to be stable?  
   [may 2013]
   An LTI system is stable if the impulse response is absolutely integrable.
   \[ \int_{-\infty}^{\infty} |h(t)| < \infty \]

9. What is meant by impulse response of any system?  
   [MAY-11]
   When the unit impulse function is applied as input to the system, the output is nothing but impulse response \( h(t) \). The impulse response is used to study various properties of the system such as causality, stability, dynamicity etc.

10. Determine the response of the system with impulse response \( h(t) = t u(t) \) for the input \( x(t) = u(t) \)  
   [DEC-11]
   The response is given as,
   \[
   y(t) = \int_{-\infty}^{\infty} h(\tau) u(t - \tau) \, d\tau.
   \]
   \[
   y(t) = \int_{-\infty}^{\infty} \tau u(\tau) u(t - \tau) \, d\tau.
   \]
   Here \( u(\tau) u(t - \tau) = 1 \) for 0 to \( t \). hence above equation will be,
   \[
   y(t) = \int_{0}^{t} \tau \, d\tau = \frac{1}{2} t^2.
   \]

11. State the properties of convolution.  
    [DEC 2009].
    1) Commutative property: \( x(t) * h(t) = h(t) * x(t) \)
    2) Associative property: \( [x(t) * h_1(t)] * h_2(t) = x(t) *[h_1(t) * h_2(t)] \)
    3) Distributive property: \( x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t) \)

12. What are the three elementary operations in block diagram representation of continuous time system?  
   [dec 2012, 2013]
   • Scalar multiplication
   \[ X(t) \rightarrow y(t) = ax(t) \]
UNIT IV

ANALYSIS OF DISCRETE TIME SIGNALS

4.1 Sampling theory

Let $x(t)$ be a continuous signal which is to be sampled, and that sampling is performed by measuring the value of the continuous signal every $T$ seconds, which is called the sampling interval. Thus, the sampled signal $x[n]$ given by: $x[n] = x(nT)$, with $n = 0, 1, 2, 3, ...$

The sampling frequency or sampling rate $f_s$ is defined as the number of samples obtained in one second, or $f_s = 1/T$. The sampling rate is measured in hertz or in samples per second.

The frequency equal to one-half of the sampling rate is therefore a bound on the highest frequency that can be unambiguously represented by the sampled signal. This frequency (half the sampling rate) is called the Nyquist frequency of the sampling system. Frequencies above the Nyquist frequency $f_N$ can be observed in the sampled signal, but their frequency is ambiguous. That is, a frequency component with frequency $f$ cannot be distinguished from other components with frequencies $Nf_N + f$ and $Nf_N - f$ for nonzero integers $N$. This ambiguity is called aliasing. To handle this problem as gracefully as possible, most analog signals are filtered with an anti-aliasing filter (usually a low-pass filter with cutoff near the Nyquist frequency) before conversion to the sampled discrete representation.

- The theory of taking discrete sample values (grid of color pixels) from functions defined over continuous domains (incident radiance defined over the film plane) and then using those samples to reconstruct new functions that are similar to the original (reconstruction).

- **Sampler**: selects sample points on the image plane
- **Filter**: blends multiple samples together

- Sampling theory
  Sampling Theorem: bandlimited signal can be reconstructed exactly if it is sampled at a rate atleast twice the maximum frequencycomponent in it."
Consider a signal $g(t)$ that is bandlimited.

**Sampling theory**

\[
III(x) = \sum_{n=-\infty}^{\infty} \delta(x - nT)
\]

\[
III(s) = \sum_{n=-\infty}^{\infty} \delta(s - ns_0)
\]

The maximum frequency component of $g(t)$ is $f_m$. To recover the signal $g(t)$ exactly from its samples it has to be sampled at a rate $f_s \geq 2f_m$. The minimum required sampling rate $f_s = 2f_m$ is called the Nyquist rate.

A continuous time signal can be processed by processing its samples through a discrete time system. For reconstructing the continuous time signal from its discrete time samples without any error, the signal should be sampled at a sufficient rate that is determined by the sampling theorem.

### 4.2 Aliasing

Aliasing is a phenomenon where the high frequency components of the sampled signal interfere with each other because of inadequate sampling $\omega_s < 2\omega_m$. Aliasing leads to distortion in the recovered signal. This is the reason why the sampling frequency should be at least twice the bandwidth of the signal.
4.3 Sampling of Non-bandlimited Signal: Anti-aliasing Filter

Anti aliasing filter is a filter which is used before a signal sampler, to restrict the
bandwidth of a signal to approximately satisfy the sampling theorem. The potential defectors
are all the frequency components beyond \( f_s/2 \) Hz. We should have to eliminate these
components from \( x(t) \) before sampling \( x(t) \). As a result of this we lose only the components
beyond the folding frequency \( f_s/2 \) Hz. These frequency components cannot reappear to
corrupt the components with frequencies below the folding frequency. This suppression of
higher frequencies can be accomplished by an ideal filter of bandwidth \( f_s/2 \) Hz. This filter is
called the **anti-aliasing filter**. The anti aliasing operation must be performed before the
signal is sampled. The anti aliasing filter, being an ideal filter is unrealizable. In practice, we
use a steep cutoff filter, which leaves a sharply attenuated residual spectrum beyond the
folding frequency \( f_s/2 \).

4.4 DISCRETE TIME FOURIER TRANSFORM

In mathematics, the discrete-time Fourier transform (DTFT) is one of the specific
forms of Fourier analysis. As such, it transforms one function into another, which is called
the frequency domain representation, or simply the "DTFT", of the original function (which
is often a function in the time-domain). But the DTFT requires an input function that is
discrete. Such inputs are often created by sampling a continuous function, like a person's
voice.

Given a discrete set of real or complex numbers: \( x[n] \), \( n \in \mathbb{Z} \) (integers), the
discrete-time Fourier transform (or DTFT) of \( x[n] \) is usually written:

\[
X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-i\omega n}.
\]

Often the \( x[n] \) sequence represents the values (aka samples) of a continuous-time
function, \( x(t) \), at discrete moments in time: \( t = nT \), where \( T \) is the sampling interval
(in seconds), and \( 1/T = f_s \) the sampling rate (samples per second). Then the DTFT
provides an approximation of the continuous-time Fourier transform:

\[
X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i2\pi ft} dt.
\]

To understand this, consider the Poisson summation formula, which indicates that a
periodic summation of function \( X(f) \) can be constructed from the samples of function
\( x(t) \). The result is:

\[
X_T(f) \overset{\text{def}}{=} \sum_{k=-\infty}^{\infty} X(f - kf_s) \equiv T \sum_{n=-\infty}^{\infty} x(nT) e^{-i2\pi fTn} = \text{Eq.2}
\]
The right-hand sides of Eq.2 and Eq.1 are identical with these associations:

\[ x[n] = T \cdot x(nT) \]
\[ \omega = 2\pi fT = 2\pi \left( \frac{f}{f_s} \right). \]

\[ X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-i\omega n}. \]

Often the \( x[n] \) sequence represents the values (aka samples) of a continuous-time function, \( x(t) \), at discrete moments in time: \( t = nT \), where \( T \) is the sampling interval (in seconds), and \( 1/T = f_s \) is the sampling rate (samples per second). Then the DTFT provides an approximation of the continuous-time Fourier transform:

\[ X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i2\pi ft} \, dt. \]

To understand this, consider the Poisson summation formula, which indicates that a periodic summation of function \( X(f) \) can be constructed from the samples of function \( x(t) \). The result is:

\[ X_T(f) \stackrel{\text{def}}{=} \sum_{k=-\infty}^{\infty} X(f - kf_s) \equiv T \sum_{n=-\infty}^{\infty} x(nT) e^{-i2\pi fTn}. \quad \text{(Eq.2)} \]

The right-hand sides of Eq.2 and Eq.1 are identical with these associations:

\[ x[n] = T \cdot x(nT) \]
\[ \omega = 2\pi fT = 2\pi \left( \frac{f}{f_s} \right). \]

\( X_T(f) \) comprises exact copies of \( X(f) \) that are shifted by multiples of \( f_s \) and combined by addition. For sufficiently large \( f_s \), the \( k=0 \) term can be observed in the region \([-f_s/2, f_s/2]\) with little or no distortion (aliasing) from the other terms.

4.5 **Inverse transform**

The following inverse transforms recover the discrete-time sequence:
\[
x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) \cdot e^{i\omega n} \, d\omega
\]

\[
= T \int_{-\frac{\pi}{2T}}^{\frac{\pi}{2T}} X_T(f) \cdot e^{i2\pi fnT} \, df.
\]

The integrals span one full period of the DTFT, which means that the \(x[n]\) samples are also the coefficients of a Fourier series expansion of the DTFT.

Infinite limits of integration change the transform into a continuous-time Fourier transform [inverse], which produces a sequence of Dirac impulses. That is:

\[
\int_{-\infty}^{\infty} X_T(f) \cdot e^{i2\pi ft} \, df = \int_{-\infty}^{\infty} \left( T \sum_{n=-\infty}^{\infty} x(nT) \cdot e^{-i2\pi fnT} \right) \cdot e^{i2\pi ft} \, df
\]

\[
= \sum_{n=-\infty}^{\infty} T \cdot x(nT) \int_{-\infty}^{\infty} e^{-i2\pi fnT} \cdot e^{i2\pi ft} \, df
\]

\[
= \sum_{n=-\infty}^{\infty} x[n] \cdot \delta(t - nT).
\]

### 4.6 Properties

- \(x[n]^*\) is the convolution between two signals
- \(x[n]^*\) is the complex conjugate of the function \(x[n]\)
- \(\rho_x[n] = \sum_{n=-\infty}^{\infty} x[n] \cdot y[n]\) represents the correlation between \(x[n]\) and \(y[n]\).
### 4.7 SYMMETRY PROPERTIES

The Fourier Transform can be decomposed into a real and imaginary part or into an even and odd part.

\[
X(e^{i\omega}) = X_R(e^{i\omega}) + iX_I(e^{i\omega})
\]

or

\[
X(e^{i\omega}) = X_E(e^{i\omega}) + X_O(e^{i\omega})
\]
4.8 *Z*-transforms

**Definition:** The *Z*– transform of a discrete-time signal *x(n)* is defined as the power series:

\[ X(z) = \sum_{k=-\infty}^{\infty} x(n)z^{-k} \]

where *z* is a complex variable. The above given relations are sometimes called the **direct Z-transform** because they transform the time-domain signal *x(n)* into its complex-plane representation *X(z)*. Since *Z*– transform is an infinite power series, it exists only for those values of *z* for which this series converges.

The **region of convergence** of *X(z)* is the set of all values of *z* for which *X(z)* attains a finite value.

**Definition:** For discrete-time systems, *z*-transforms play the same role of Laplace transforms do in continuous-time systems.

**Bilateral forward Z transform**

\[ H[z] = \sum_{n=-\infty}^{\infty} h[n]z^{-n} \]

**Bilateral inverse Z transform**

\[ h[n] = \frac{1}{2\pi j} \oint_{C} H[z]z^{n+1}dz \]
**Z-transform Pairs**

\[ h[n] = d[n] \]

Region of convergence: entire \( z \)-plane

\[ H[z] = \sum_{n=-\infty}^{\infty} \delta[n]z^{-n} = \sum_{n=0}^{0} \delta[n]z^{-n} = 1 \]

\[ h[n] = d[n-1] \]

Region of convergence: entire \( z \)-plane

\[ h[n-1] \iff z^{-1} H[z] \]

\[ H[z] = \sum_{n=-\infty}^{\infty} \delta[n-1]z^{-n} = \sum_{n=1}^{1} \delta[n-1]z^{-n} = z^{-1} \]

**Inverse z-transform**

\[ f[n] = \frac{1}{2\pi j} \oint_{c-j\infty}^{c+j\infty} F[z]z^{-n-1}dz \]

Using the definition requires a contour integration in the complex \( z \)-plane.

Fortunately, we tend to be interested in only a few basic signals (pulse, step, etc.) Virtually all of the signals we’ll see can be built up from these basic signals.

**4.9 Z transform properties**

**Z-transform Properties**

Properties of \( z \)-transform

1. Linearity

\[ Z(x_1(nT) + x_2(nT)) = Z(x_1(nT)) + Z(x_2(nT)) \]

2. Initial Value

\[ x(0) = \lim_{z \to \infty} X(z) \]

\[ X(z) = x(0) + x(1)z^{-1} + \cdots \]

3. Final value

\[ x(\infty) = \lim_{z \to 1} (1 - z^{-1}).X(z) \]

\[ x(\infty) = \lim_{s \to 0} sX(s) \]
\[
\begin{align*}
\frac{1}{s} & \quad \leftrightarrow \quad \frac{1}{1-z^{-1}} \\
\lim_{s \to 0} sX(s) & \quad \leftrightarrow \quad \lim_{z \to 1} (1-z^{-1})X(z)
\end{align*}
\]

1. **Periodicity:**
\[X(e^{j(\omega+2\pi)}) = X(e^{j\omega})\]

2. **Linearity:**
\[ax_1[n] + bx_2[n] \quad \leftrightarrow \quad aX_1(e^{j\omega}) + bX_2(e^{j\omega})\]

3. **Time Shift:**
\[x[n-n_0] \quad \leftrightarrow \quad e^{-j\omega n_0}X(e^{j\omega})\]

4. **Phase Shift:**
\[e^{j\omega n_0}x[n] \quad \leftrightarrow \quad X(e^{j(\omega-\omega_0)})\]

5. **Conjugacy:**
\[x^*[n] \quad \leftrightarrow \quad X^*(e^{-j\omega})\]

6. **Time Reversal**
\[x[-n] \quad \leftrightarrow \quad X(e^{-j\omega})\]

7. **Differentiation**
\[nx[n] \quad \leftrightarrow \quad j\frac{dX(e^{j\omega})}{d\omega}\]

8. **Parseval Equality**
\[\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega\]

9. **Convolution**
\[y[n] = x[n] * h[n] \quad \leftrightarrow \quad Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})\]

10. **Multiplication**
\[y[n] = x_1[n]x_2[n] \quad \leftrightarrow \quad Y(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\omega})X_2(e^{j(\omega-\theta)})d\theta\]
Sample Problem:

1. Obtain the z transform of,

\[ X(z) = \frac{1}{z^2(z-0.5)} \]

We expand \( X(z)/z \) into simple fractions as

\[
\frac{X(z)}{z} = \frac{1}{z^2(z-0.5)} = \frac{K_1}{z^3} + \frac{K_2}{z^2} + \frac{K_3}{z} + \frac{K_4}{z-0.5}
\]

where

\[
K_1 = \left. \frac{1}{z^3} \frac{X(z)}{z} \right|_{z=0} = \frac{1}{z-0.5} \big|_{z=0} = -2
\]
\[
K_2 = \left. \frac{d}{dz} \frac{1}{z^2} X(z) \right|_{z=0} = \frac{d}{dz} \frac{1}{z-0.5} \big|_{z=0} = \frac{1}{z-0.5} \big|_{z=0} = -4
\]
\[
K_3 = \left. \frac{d^2}{dz^2} \frac{1}{z^2} X(z) \right|_{z=0} = \frac{d}{dz} \frac{1}{2(z-0.5)^2} \big|_{z=0} = \frac{1}{2} \frac{1}{(z-0.5)^2} \big|_{z=0} = -8
\]
\[
K_4 = \left. (z-0.5) \frac{1}{z^2} \frac{X(z)}{z} \right|_{z=0.5} = \frac{1}{z^3} \big|_{z=0.5} = 8
\]

Thus, \( X(z) \) is expanded as

\[
X(z) = -2z^{-2} - 4z^{-1} - 8 + \frac{8}{1-0.5z^{-1}}
\]

2. Find the inverse z transform of,

\[ X(z) = \frac{z^2 + z + 2}{(z-1)(z^2 - z + 1)} \]

by use of the partial-fraction expansion method.

With complex conjugate poles \((z_{2,3} = 0.5 \pm j0.866\) with \(|z_{2,3}| = 1\) in the quadratic factor \(z^2 - z + 1\), we expand \(X(z)\) in simple partial fractions as

\[
X(z) = \frac{4}{z-1} + \frac{-3z + 2}{z^2 - z + 1} \text{ or } X(z) = \frac{4z^{-1}}{1-z^{-1}} + \frac{-3z^{-1} + 2z^{-2}}{1-z^{-1}+z^{-2}}
\]

Recalling that the z transform of damped cosine and sine functions are given by

\[
X[\cos(\omega T)] = \frac{1 - e^{-aT}z^{-1} \cos \omega T}{1 - 2e^{-aT}z^{-1} \cos \omega T + e^{-2aT}z^{-2}}
\]
\[
X[\sin(\omega T)] = \frac{e^{-aT}z^{-1} \sin \omega T}{1 - 2e^{-aT}z^{-1} \cos \omega T + e^{-2aT}z^{-2}},
\]

we observe that the second expanded term in the expression of \(X(z)\) above can be viewed as the z transform of a damped sinusoid. Actually, \(X(z)\) can be rewritten as

\[
X(z) = \frac{4z^{-1}}{1-z^{-1}} - 3 \left( \frac{z^{-1} - 0.5z^{-2}}{1-z^{-1}+z^{-2}} \right) + \frac{0.5z^{-2}}{1-z^{-1}+z^{-2}}
\]
\[
= \frac{4z^{-1}}{1-z^{-1}} - 3z^{-1} - \frac{1-0.5z^{-1}}{1-z^{-1}+z^{-2}} + z^{-1} \frac{0.5z^{-1}}{1-z^{-1}+z^{-2}}
\]
2 mark questions

1. What is the relation between Z transform and fourier transform of discrete time signal. (APR/MAY 2010).

\[ X(\omega) = X(Z)|_{z = e^{j\omega}} \]  

This means Z transform is same as fourier transform when evaluated on unit circle.

2. Define region of convergence with respect to Z transform. [MAY-11, 2015].

Region of convergence (ROC) is the area in Z plane where Z transform convergence. In other word, it is possible to calculate the \( X(z) \) in ROC.

3. State the initial value theorem of Z transforms. (APR/MAY 2010).

The initial value of the sequence is given as, \( X(0) = \lim_{z \to 1} X(z) \).

4. What is meant by aliasing? (MAY/JUN 2010).

When the high frequency interferes with low frequency and appears as low then the phenomenon is called aliasing.

5. Define Nyquist rate and Nyquist interval. (MAY/JUN 2010).

When the sampling rate becomes exactly equal to ‘2W’ samples/sec, for a give bandwidth of W hertz, then it is called Nyquist rate.’ 

Nyquist interval is the time interval between any two adjacent samples.

Nyquist rate =2W hz & Nyquist interval=1/2W seconds.

6. Define unilateral Z-Transform or one sided Z-transform [MAY-10]

The unilateral Z-Transform of signal \( x(t) \) is given as,

\[ X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \]

The unilateral and bilateral Z-Transforms are same for causal signals.

7. State the final value theorem for z-transform. [may 2012]

The final value of a sequence is given as,

\[ x(\infty) = \lim_{z \to 1} (1 - z^{-1})X(z) \]
8. Define DTFT pair. [dec 2012]

DTFT,
\[ X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \]  

(analysis equation)

\[ x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega \]  

(Synthesis equation)

9. State the sampling theorem.

- A bandwidth signal of finite energy, which has no frequency components higher than \( W \) hertz, is completely described by specifying the values of the signal at instants of time separated by \( 1/2W \) seconds.
- A band limited signal of finite energy, which has no frequency components higher than \( W \) hertz, may be completely recovered from the knowledge of its samples taken at the rate of \( 2W \) samples per second.

10. Define two sided Z transform. [may 2010, 2013]

The z-transform of the DT signal is given by,
\[ X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \]

Here ‘z’ is the complex variable. The z-transform pair is denoted by,
\[ x(n) \leftrightarrow X(Z) \]
\[ y(t) = \int_{-\infty}^{t} x(\tau) \]

11. State the convolution property of z transform. [dec 2012]

The convolution states that,
If \( x_1(n) \leftrightarrow X_1(z) \)
\( x_2(n) \leftrightarrow X_2(z) \)
then \( x_1(n) * x_2(n) \leftrightarrow X_1(z)X_2(z) \)

That is the convolution of two sequences in time domain is equivalent to multiplication of their z-transforms.

12. State Parseval’s theorem.

Consider the complex valued sequences \( x(n) \) and \( y(n) \). If
\( x(n) \leftrightarrow X(k) \)
\( y(n) \leftrightarrow Y(k) \)
then \( x(n)y^*(n) = 1/N \sum_{k=-\infty}^{\infty} X(k)Y^*(k) \)

13. Find Z transform of \( x(n) = \{1,2,3,4\} \)
\[ x(n) = \{1,2,3,4\} \]
\[ X(z) = x(n)z^{-n} \]
\[= 1+2z^{-1}+3z^{-2}+4z^{-3}.\]
\[= 1+2/z+3/z^2+4/z^3.\]

**14. What z transform of \((n-m)\)?**
By time shifting property
\[Z[A(n-m)] = AZ-m \sin Z[n] = 1\]

**15. Obtain the inverse z transform of \(X(z) = 1/z-a, |z|>|a|\)**
Given \(X(z) = z-1/1-az-1\)
By time shifting property
\[X(n) = a \cdot u(n-1)\]
5.1 Introduction

A discrete-time system is anything that takes a discrete-time signal as input and generates a discrete-time signal as output. The concept of a system is very general. It may be used to model the response of an audio equalizer. In electrical engineering, continuous-time signals are usually processed by electrical circuits described by differential equations.

For example, any circuit of resistors, capacitors and inductors can be analyzed using mesh analysis to yield a system of differential equations. The voltages and currents in the circuit may then be computed by solving the equations. The processing of discrete-time signals is performed by discrete-time systems. Similar to the continuous-time case, we may represent a discrete-time system either by a set of difference equations or by a block diagram of its implementation.

For example, consider the following difference equation. \( y(n) = y(n-1) + x(n) + x(n-1) + x(n-2) \) This equation represents a discrete-time system. It operates on the input signal \( x(n) \) to produce the output signal \( y(n) \).

5.2 BLOCK DIAGRAM REPRESENTATION

Block diagram representation of

\[
y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n]
\]

LTI systems with rational system function can be represented as constant-coefficient difference equation
• The implementation of difference equations requires delayed values of the
  – input
  – output
  – intermediate results
• The requirement of delayed elements implies need for storage

![Difference Equation Diagram]

• We also need means of
  – addition
  – multiplication

**Direct Form I**

General form of difference equation

\[
\sum_{k=0}^{N} \hat{a}_k y[n-k] = \sum_{k=0}^{M} \hat{b}_k x[n-k]
\]

Alternative equivalent form
• Cascade form

General form for cascade implementation

\[ H(z) = A \frac{\prod_{k=1}^{M_1} (1 - f_k z^{-1}) \prod_{k=1}^{M_2} (1 - g_k z^{-1}) (1 - g_k^* z^{-1})}{\prod_{k=1}^{N_1} (1 - c_k z^{-1}) \prod_{k=1}^{N_2} (1 - d_k z^{-1}) (1 - d_k^* z^{-1})} \]
5.3 CONVOLUTION SUM

The convolution sum provides a concise, mathematical way to express the output of an LTI system based on an arbitrary discrete-time input signal and the system's response. The convolution sum is expressed as,

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]

- Convolution is commutative
  \[ x[n] * h[n] = h[n] * x[n] \]
- Convolution is distributive
  \[ x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n] \]

- Cascade connection:
  \[ y[n] = h_1[n] * (h_2[n] * x[n]) = (h_1[n] * h_2[n]) * x[n] \]

- Parallel connection
  \[ y[n] = h_1[n] * x[n] + h_2[n] * x[n] = (h_1[n] + h_2[n]) * x[n] \]

- LTI systems are stable if
  \[ \sum_{k=-\infty}^{\infty} |h[k]| < \infty \]

LTI systems are causal if
\[ h[n] = 0 \quad n < 0 \]
5.4 LTI System analysis using DTFT

**LTI SYSTEMS ANALYSIS USING DTFT**

- Consider \( X(e^{j\omega}) = |X(e^{j\omega})|e^{j\angle X(e^{j\omega})} \) and \( H(e^{j\omega}) = |H(e^{j\omega})|e^{j\angle H(e^{j\omega})} \)

  - **Magnitude**
    \[ |Y(e^{j\omega})| = |X(e^{j\omega})||H(e^{j\omega})| \]

  - **Phase**
    \[ \angle Y(e^{j\omega}) = \angle X(e^{j\omega}) + \angle H(e^{j\omega}) \]

  Frequency response at \( H(e^{j\omega}) = H(z) |_{|z|=1} \) is valid if ROC includes \( |z|=1 \),

  \[ Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \]

5.5 LTI SYSTEMS ANALYSIS USING Z-TRANSFORM

- The z-transform of impulse response is called transfer or system function \( H(z) \).

  \[ Y(z) = X(z)H(z) \]

- **General form of LCCDE**

  \[ \sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \]

  \[ \sum_{k=0}^{N} a_k z^{-k}Y(z) = \sum_{k=0}^{M} b_k z^{-k}X(z) \]

  **Compute the z-transform**

  \[ H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \]
System Function: Pole/zero Factorization

- Stability requirement can be verified.
- Choice of ROC determines causality.
- Location of zeros and poles determines the frequency response and phase

\[ H(z) = \frac{b_0}{a_0} \prod_{k=1}^{M} \frac{(1 - c_k z^{-1})}{(1 - d_k z^{-1})} \]

Sample Problems:

1. Consider the system described by the difference equation.

\[ y[n] = x[n] + \frac{1}{3} x[n - 1] + \frac{5}{4} y[n - 1] - \frac{1}{2} y[n - 2] + \frac{1}{16} y[n - 3] \]

Here \( N = 3, \ M = 1 \). Order 3 homogeneous equation:

\[ y[n] - \frac{5}{4} y[n - 1] + \frac{1}{2} y[n - 2] - \frac{1}{16} y[n - 3] = 0 \quad n \geq 2 \]

The characteristic equation:

\[ 1 - \frac{5}{4} a^{-1} + \frac{1}{2} a^{-2} - \frac{1}{16} a^{-3} = 0 \]

The roots of this third order polynomial is: \( a_1 = a_3 = 1/2 \quad a_2 = 1/4 \) and

\[ y_k[n] = h[n] = a_1 \left( \frac{1}{2} \right)^n + a_2 n \left( \frac{1}{2} \right)^n + a_3 \left( \frac{1}{4} \right)^n, \quad n \geq 2 \]

Let us assume \( y[-1] = 0 \) then (3.52) for this case becomes:

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
[y[0]] \\
y[1] \\
y[2]
\end{bmatrix}
= \begin{bmatrix}
-1/4 & 0 & 1 \\
0 & 1/2 & 1/3 \\
1/3 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
[y[0]] \\
y[1] \\
y[2]
\end{bmatrix}
\]

with these we have the impulse response of this system:

\[ h[n] = -\frac{4}{3} \left( \frac{1}{2} \right)^n + \frac{10}{3} n \left( \frac{1}{2} \right)^n + \frac{7}{3} \left( \frac{1}{4} \right)^n \quad n \geq 0 \]
2. Given \( y[-1] = 1 \) and \( y[-2] = 0 \). Compute recursively a few terms of the following 2\(^{nd}\) order DE:

\[
\begin{align*}
y[n] &= \frac{3}{4} y[n-1] - \frac{1}{8} y[n-2] + (\frac{1}{2})^n \\
y[0] &= \frac{3}{4} y[-1] - \frac{1}{8} y[-2] + (\frac{1}{2})^0 - \frac{3}{4} + 0 + 1 - \frac{7}{4} \\
y[1] &= \frac{3}{4} y[0] - \frac{1}{8} y[-1] + (\frac{1}{2})^1 = \frac{27}{16} \\
y[2] &= \frac{3}{4} y[1] - \frac{1}{8} y[0] + (\frac{1}{2})^2 = \frac{83}{64} \\
&\vdots
\end{align*}
\]

3. Compute the impulse response of the system described by,

\[
y[n] - \frac{1}{2} y[n-1] = x[n].
\]

Solution: if \( x[n] = \delta[n] \), then \( y[n] = h[n] \) is the impulse response.

\[
y[n] = \frac{1}{2} y[n-1] + x[n]
\]

\[
\Rightarrow h[n] = \frac{1}{2} h[n-1] + \delta[n]
\]

\[
h[0] = \frac{1}{2} h[-1] + \delta[0]
\]

If we assume condition of initial rest \( h[-1] = 0 \),

then

\[
h[0] = 1
\]

\[
h[1] = \frac{1}{2} h[0] + \delta[1] = \frac{1}{2} + 0 = \frac{1}{2}
\]

\[
h[2] = \frac{1}{2} h[1] + \delta[2] = (\frac{1}{2})^2
\]

\[
\vdots
\]

\[
h[n] = (\frac{1}{2})^n, \text{ for } n \geq 0
\]

\[
h[n] = 0, \text{ for } n < 0
\]

\[
\Rightarrow h[n] = (\frac{1}{2})^n u[n]
\]

- The response of the system is not limited to a finite time interval. This is called an infinite impulse response (IIR) system.
4. Obtain the structures realization of LTI system

\[ y[n] = -a_1 y[n-1] + v[n] \]
\[ v[n] = b_0 x[n] + b_1 x[n-1] \]

\[ w[n] = -a_1 w[n-1] + x[n] \]
\[ y[n] = b_0 w[n] + b_1 w[n-1] \]

Direct Form I

Direct Form II

Generalizes to higher order systems described by difference equations.

\[ y[n] = - \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \]

\[ v[n] = \sum_{k=0}^{M} b_k x[n-k] \]

\[ y[n] = - \sum_{k=1}^{N} a_k y[n-k] + v[n] \]
The first system $v[n] = \ldots$ is nonrecursive, where as the second system is recursive.

![Diagram](image)

**Figure 2.3: General Direct Form I.**

\[
w[n] = - \sum_{k=1}^{N} a_k w[n-k] + x[n]
\]
\[
y[n] = \sum_{k=0}^{M} b_k w[n-k]
\]
5. Find the convolution of \(x(n)=[1,1,1,1,2,2,2,2]\) with \(h(n)=[3,3,0,0,0,0,3,3]\) by using matrix method.

Solution: By using matrix method, \(N=8\)

\[
\begin{bmatrix}
  y(0) \\
  y(1) \\
  y(2) \\
  y(3) \\
  y(4) \\
  y(5) \\
  y(6) \\
  y(7)
\end{bmatrix} = 
\begin{bmatrix}
  h(0) & h(7) & h(6) & h(5) & h(4) & h(3) & h(2) & h(1) \\
  h(1) & h(0) & h(7) & h(6) & h(5) & h(4) & h(3) & h(2) \\
  h(2) & h(1) & h(0) & h(7) & h(6) & h(5) & h(4) & h(3) \\
  h(3) & h(2) & h(1) & h(0) & h(7) & h(6) & h(5) & h(4) \\
  h(4) & h(3) & h(2) & h(1) & h(0) & h(7) & h(6) & h(5) \\
  h(5) & h(4) & h(3) & h(2) & h(1) & h(0) & h(7) & h(6) \\
  h(6) & h(5) & h(4) & h(3) & h(2) & h(1) & h(0) & h(7) \\
  h(7) & h(6) & h(5) & h(4) & h(3) & h(2) & h(1) & h(0)
\end{bmatrix} 
\begin{bmatrix}
  x(0) \\
  x(1) \\
  x(2) \\
  x(3) \\
  x(4) \\
  x(5) \\
  x(6) \\
  x(7)
\end{bmatrix}
\]

Substituting the values, we get

\[
\begin{bmatrix}
  y(0) \\
  y(1) \\
  y(2) \\
  y(3) \\
  y(4) \\
  y(5) \\
  y(6) \\
  y(7)
\end{bmatrix} = 
\begin{bmatrix}
  3 & 3 & 3 & 0 & 0 & 0 & 0 & 3 \\
  3 & 3 & 3 & 3 & 0 & 0 & 0 & 1 \\
  0 & 3 & 3 & 3 & 3 & 0 & 0 & 0 \\
  0 & 0 & 3 & 3 & 3 & 3 & 0 & 0 \\
  0 & 0 & 0 & 3 & 3 & 3 & 3 & 0 \\
  0 & 0 & 0 & 0 & 3 & 3 & 3 & 3 \\
  3 & 0 & 0 & 0 & 0 & 3 & 3 & 3 \\
  3 & 3 & 0 & 0 & 0 & 0 & 3 & 3
\end{bmatrix} 
\begin{bmatrix}
  1 \\
  1 \\
  1 \\
  1 \\
  2 \\
  2 \\
  2 \\
  2
\end{bmatrix}
\]

\[
\begin{bmatrix}
  y(0) \\
  y(1) \\
  y(2) \\
  y(3) \\
  y(4) \\
  y(5) \\
  y(6) \\
  y(7)
\end{bmatrix} = 
\begin{bmatrix}
  3\times1 + 3\times1 + 3\times1 + 0\times1 + 0\times2 + 0\times2 + 0\times2 + 3\times2 \\
  3\times1 + 3\times1 + 3\times1 + 3\times1 + 0\times2 + 0\times2 + 0\times2 + 0\times2 \\
  0\times1 + 3\times1 + 3\times1 + 3\times1 + 2\times0 + 2\times0 + 2\times0 + 2 \\
  0\times1 + 0\times1 + 3\times1 + 3\times1 + 3\times2 + 3\times2 + 0\times2 + 0\times2 \\
  0\times1 + 0\times1 + 0\times1 + 3\times1 + 3\times2 + 3\times2 + 3\times2 + 0\times2 \\
  0\times1 + 0\times1 + 0\times1 + 0\times1 + 3\times2 + 3\times2 + 3\times2 + 3\times2 \\
  3\times1 + 0\times1 + 0\times1 + 0\times1 + 0\times2 + 3\times2 + 3\times2 + 3\times2 \\
  3\times1 + 3\times1 + 0\times1 + 0\times1 + 0\times2 + 0\times2 + 3\times2 + 3\times2
\end{bmatrix} 
\begin{bmatrix}
  15 \\
  12 \\
  15 \\
  18 \\
  21 \\
  24 \\
  21 \\
  18
\end{bmatrix}
\]

Therefore, the convoluted sum is \(y(n) = [15, 12, 15, 18, 21, 24, 21, 18]\).
2 mark questions and answers

1. States the properties of convolution. (DEC 2009).
   i). Commutative property of convolution
   \[ x(n) * h(n) = h(n) * x(n) = y(n) \]
   ii). Associative property of convolution
   \[ [x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)] \]
   iii). Distributive property of convolution
   \[ x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n). \]

2. Define non recursive and recursive of the following system. (MAY/JUN 2010).
   When the output \( y(n) \) of the system depends upon present and past inputs then it is called non-recursive system. When the output \( y(n) \) of the system depends upon present and past inputs as well as past outputs, then it is called recursive system.

3. Define convolution sum?
   If \( x(n) \) and \( h(n) \) are discrete variable functions, then its convolution sum \( y(n) \) is given by,
   \[ y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) \]

4. If \( x(n) \) and \( y(n) \) are discrete variable functions, what is its convolution sum. [dec 2013]
   The convolution sum is,
   \[ \sum_{k=-\infty}^{\infty} x(k) y(n-k) \]

5. Determine the system function of the discrete time system described by the difference equation.
   \[ Y(n) = 0.5y(n-1) + x(n) \] [may 2012]
   Taking z-transform of both sides,
   \[ Y(z) = 0.5z^{-1}Y(z) + X(z) \]
   \[ H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.5z^{-1}} \]

6. A causal LTI system has impulse response \( h(n) \), for which the z-transform is \( H(z) = \frac{1+z^{-1}}{(1-0.5z^{-1})(1+0.25z^{-1})} \). Is the system stable? Explain.
   \( H(z) \) can be written in terms of positive powers of \( z \) as follows:
   \[ H(z) = \frac{z(z+1)}{(z-0.5)(z+0.25)} \]
Poles are at $p_1 = 0.5$ and $p_2 = -0.25$. Since both the poles are inside unit circle, this system is stable.

7. Check whether the system with system function $H(Z) = (1/1-0.5z^{-1})+(1/1-2z^{-1})$ with ROC $|z| < 0.5$ is causal and stable? [dec 2013]

$H(z) = z/(z - 0.5) + z/(z - 2)$. Poles of this system are located at $z = 0.5$ and $z = 2$. This system is not causal and stable, since all poles are not located inside unit circle.

8. Is the discrete time system described by the difference equation $y(n) = x(-n)$ is causal? [may 2013]

Here $y(-2) = x(-(-2)) = x(2)$. This means output at $n = -2$ depends upon future inputs. Hence this system is not causal.

9. Consider a system whose impulse is $h(t) = e^{|t|}$. Is this system causal or non causal? [dec 2011]

Here $h(t) = e^{-t}$ for $t \geq 0$  
$= e^{t}$ for $t < 0$

Since $h(t)$ is not equal to zero for $t < 0$, the system is non causal.

10. Find the step response of the system if the impulse response $h(n) = \delta(n - 2) - \delta(n - 1)$ [may 2011]

Solution:
$Y(n) = h(n)*u(n)$, since $x(n) = u(n)$, step input.

$= \delta(n - 2) * u(n) - \delta(n - 1) * u(n)$

$= u(n - 2) - u(n - 1)$

11. Obtain the convolution of
   a) $X(n) * \delta(n)$
   b) $X(n) * [h_1(n) + h_2(n)]$

Solution:
$x(n)* \delta(n) = \delta(n)$

$x(n)*[h_1(n) + h_2(2)] = x(n)*h_1(n) + x(n)*h_2(n)$

12. List the steps involved in finding convolution sum?
   o folding
   o Shifting
   o Multiplication
   o Summation
13. Consider an LTI system with impulse response $h(n) = \delta(n-n_0)$ for an input $x(n)$, find the $Y(e^{j\omega})$. (NOV/DEC 2003).

Here is the spectrum of output. By convolution theorem we can write,

\[ Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) \]

Here \[ H(e^{j\omega}) = \text{DTFT}\{\delta(n-n_0)\} = e^{-j\omega n_0} \]
\[ Y(e^{j\omega}) = e^{-j\omega n_0} X(e^{j\omega}) \]

14. List the properties of convolution?

- **Commutative property of convolution**
  \[ x(n) * h(n) = h(n) * x(n) = y(n) \]

- **Associative property of convolution**
  \[ [x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)] \]

- **Distributive property of convolution**
  \[ x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n) \]

15. Define system function?

$H(z) = Y(z)$ is called system function. It is the $z$ transform of the unit sample $X(Z)$ response $h(n)$ of the system.
Question Paper Code: 11281

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2011
Third Semester
Electronics and Communication Engineering
EC 2204 — SIGNALS AND SYSTEMS
(Regulation 2008)

Time: Three hours
Maximum: 100 marks

Answer ALL questions

PART A — (10 x 2 = 20 marks)

1. Define step and impulse function in discrete signals.
2. Distinguish between deterministic and random signals.
3. Obtain the Fourier series coefficients for \( x(n) = \sin\frac{n\pi}{4} \).
4. State the initial and final value theorem of Laplace transforms.
5. What is meant by impulse response of any system?
6. Define convolution integral of continuous time systems.
7. Define sampling theorem.
8. What is ROC in Z transforms?
9. Obtain the convolution of
   (a) \( x(n) * \delta(n) \)
   (b) \( x(n) * [h_1(n) + h_2(n)] \).
10. Write the difference equation for non recursive system.

PART B — (5 x 16 = 80 marks)

11. (a) (i) Check the following for linearity, time invariance causality and stability:
    \( y(n) = x(n) + nx(n + 1) \).

    (ii) Check whether the following are periodic:
    (1) \( x(n) = \sin\left(\frac{6\pi}{7}n + 1\right) \)
    (2) \( x(n) = e^{j\frac{\pi}{5}(n + \frac{1}{2})} \).

    Or
(b) ii) Sketch the following signals:

1. \( x(t) = r(t) \)
2. \( x(t) = r(-t + 2) \)
3. \( x(t) = -2r(t) \)

where \( r(t) \) is a Ramp signal.

12. (a) i) Distinguish between Fourier series analysis and Fourier transforms.

(ii) Obtain the Fourier series of the following half wave rectified sine wave.

![Graph of half wave rectified sine wave]

Or

(b) i) Find the Laplace transform of the following:

1. \( x(t) = u(t - 2) \)
2. \( x(t) = t^2 e^{-2t} u(t) \)

(ii) Find the step response of the following circuit using Laplace transform.

![Circuit diagram]

13. (a) i) Realize the following indirect form II

\[
\frac{d^3 y(t)}{dt^3} + 4 \frac{d^2 y(t)}{dt^2} + 7 \frac{dy(t)}{dt} + 8y(t) = 5 \frac{d^2 x(t)}{dt^2} + 4 \frac{dx(t)}{dt} + 7x(t).
\]

(ii) Obtain the convolution of the signals \( x_1(t) = e^{-\alpha t} u(t) \), \( x_2(t) = e^{-\beta t} u(t) \) using Fourier transform.

Or

www.vidyarthiplus.com
(b) \( \frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 8y(t) - 2x(t) \)

Find impulse response of the system. \( (8) \)

(ii) Explain state space representation of the system. \( (8) \)

14. (a) 

(i) What is aliasing? Explain with an example. \( (6) \)

(ii) Obtain the inverse \( s \) transform of \( X(Z) = \frac{Z^2}{Z^2 - 1.52 + 0.5} \) for ROC

\[
\begin{align*}
(1) & \quad |Z| > 1 \\
(2) & \quad |Z| < 0.5 \\
(3) & \quad 0.5 < |Z| < 1.
\end{align*}
\]

Or

(b) State and explain the following properties of DTFT:

(i) Convolution property

(ii) Time shifting

(iii) Time reversal

(iv) Frequency shifting. \( (16) \)

15. (a) 

(i) Obtain the cascade realization of
\[ y(n) = \frac{1}{4} y(n-1) - \frac{1}{8} y(n-2) - x(n) + 3x(n-1) + 2x(n-2). \]

(ii) Obtain the relationship between DTFT and \( Z \) transforms. \( (4) \)

Or

(b) The state variable description of the system is given as
\[ A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 0 \end{bmatrix}, \quad D = [b] \]

Determine the transfer function of the system. \( (16) \)
B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009

Third Semester
Electronics and Communication Engineering

EC 2204 — SIGNALS AND SYSTEMS
(Regulation 2008)

Time : Three hours  Maximum : 100 Marks

Answer ALL Questions

PART A — (10 × 2 = 20 Marks)

1. For the signal shown in Fig. 1, find $x(2t + 3)$.

![Fig. 1]

2. What is the classification of the systems?

3. What are the Dirichlet’s conditions of Fourier series?

4. State convolution property of Fourier transform.

5. What is the Laplace transform of the function $x(t) = u(t) - u(t - 2)$?

6. What are the transfer functions of the following?
   (a) An ideal integrator
   (b) An ideal delay of $T$ seconds.

7. What is an anti–aliasing filter?


9. What is the z-transform of the sequence $x(n) = a^n u(n)$?

10. Define system function.
PART B — (5 × 16 = 80 Marks)

11. (a) (i) Write about elementary continuous–time signals in detail. (8)
(ii) Determine the power and RMS value of the following signals
(2 × 4 = 8)
\[ x_1(t) = 5 \cos \left( 50t + \frac{\pi}{3} \right) \]
\[ x_2(t) = 10 \cos 5t \cos 10t. \]

Or

(b) (i) Determine whether the following systems are linear or not.
(2 × 4 = 8)
\[ \frac{dy}{dt} + 3ty(t) = t^2x(t) \]
\[ y(n) = 2x(n) + \frac{1}{x(n - 1)} \]

(ii) Determine whether the following systems are time–invariant or not.
(2 × 4 = 8)
\[ y(t) = tx(t) \]
\[ y(n) = x(2n) \]

12. (a) (i) Find the trigonometric Fourier series for the periodic signal \( x(t) \)
shown in Fig. 2. (8)

(ii) Find the Fourier transform of rectangular pulse. Sketch the signal
and its Fourier transform. (8)

Or

(b) (i) Determine the Laplace transform of the following signals.
(2 × 4 = 8)
\[ x_1(t) = u(t - 2) \quad x_2(t) = t^2 e^{-2t} u(t). \]
(ii) Determine the Laplace transform of the signal. (8)
\[ x(t) = \sin \pi t; \ 0 < t < 1 \]
\[ = 0 \text{ otherwise} \]

13. (a) (i) Find the convolution of the following signals. (8)
\[ x_1(t) = e^{-at} u(t); \ x_2(t) = e^{-bt} u(t). \]
(ii) Find the impulse response of the system shown in Fig. 3.  

![Fig. 3](image)

Or

(b) For the circuit shown in Fig. 4, obtain state variable equation. The input voltage source is $x(t)$ and the output $y(t)$ is taken across capacitor $C_2$.

![Fig. 4](image)

14. (a) (i) Determine the Nyquist sampling rate and Nyquist sampling interval for the signal $x(t) = \sin e^{(200 \pi t)}$.  

(ii) Determine the z-transform of the signal $x(n) = \sin \omega_0 n u(n)$.  

Or

(b) (i) State and prove time shifting property of discrete–time Fourier Transform.  

(ii) State and prove convolution property of z-transform.

15. (a) Find the impulse and step response of the following system:  

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n).$$

Or

(b) Obtain the cascade and parallel form realization of the following system.  

$$y(n) - \frac{1}{4} y(n-1) - \frac{1}{8} y(n-2) = x(n) + 3x(n-1) + 2x(n-2).$$
PART A — (10 \times 2 = 20 Marks)

1. Define unit impulse and unit step signals.

2. When is a system said to be memoryless? Give an example.


4. Find the Laplace transform of the signal \( x(t) = e^{-at} u(t) \).

5. State the convolution integral for continuous time LTI systems.

6. What is the impulse response of two LTI systems connected in parallel?

7. State the Sampling theorem.

8. State the sufficient condition for the existence of DTFT for an aperiodic sequence \( x(n) \).


10. Define the shifting property of the discrete time unit Impulse function.
PART B — (5 × 16 = 80 Marks)

11. (a) Distinguish between the following:
   (i) Continuous Time Signal and Discrete Time Signal. (4)
   (ii) Unit step and Unit Ramp functions. (4)
   (iii) Periodic and Aperiodic signals. (4)
   (iv) Deterministic and Random signals. (4)

Or

(b) (i) Find whether the signal \( x(t) = 2\cos(10t + 1) - \sin(4t - 1) \) is periodic or not. (4)

(ii) Find the summation \( \sum_{n=-\infty}^{\infty} e^{2n} \delta(n - 2) \). (4)

(iii) Explain the properties of unit impulse function. (4)

(iv) Find the fundamental period \( T \) of the continuous time signal \( x(t) = 20\cos\left(10 \pi t + \frac{\pi}{6}\right) \). (4)

12. (a) (i) Find the trigonometric Fourier series for the periodic signal \( x(t) \) shown in the figure given below: (10)

(ii) Explain the Fourier spectrum of a periodic signal \( x(t) \). (6)

Or

(b) (i) Find the Laplace transform of the signal \( x(t) = e^{-at} u(t) + e^{-bt} u(-t) \). (8)

(ii) Find the Fourier transform of \( x(t) = e^{-|t|} \) for \(-1 \leq t \leq 1\)

\( = 0 \) otherwise. (8)
13. (a) (i) Explain the steps to compute the convolution integral. (8)

(ii) Find the convolution of the following signals: (8)

\[ x(t) = e^{-2t} u(t) \]
\[ h(t) = u(t + 2) . \]

Or

(b) (i) Using Laplace transform, find the impulse response of an LTI system described by the differential equation

\[ \frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t) . \] (8)

(ii) Explain the properties of convolution integral. (8)

14. (a) (i) Find the Fourier Transform of

\[ x(n) = A \quad |n| \leq N \]
\[ = 0 \quad |n| > N . \] (8)

(ii) Explain any four properties of DTFT. (8)

Or

(b) (i) Find the Z-transform of the given signal \( x(n) \) and find ROC.

\[ x(n) = [\sin \omega n] u(n) . \] (10)

(ii) Describe the sampling operation and explain how aliasing error can be prevented. (6)

15. (a) (i) Find the impulse response of the discrete time system described by the difference equation

\[ y(n-2) - 3y(n-1) + 2y(n) = x(n-1) . \] (8)

(ii) Discuss the block diagram representation for LTI discrete time systems. (8)

Or

(b) (i) Describe the state variable model for discrete time systems. (8)

(ii) Find the state variable matrices A, B, C, D for the equation

\[ y(n) - 3y(n-1) - 2y(n-2) = x(n) + 5x(n-1) + 6x(n-2) . \] (8)

Third Semester

Electronics and Communication Engineering

EC 6308 — SIGNALS AND SYSTEMS

(Common to Biomedical Engineering and Medical Electronics Engineering)

(Regulation 2013)

Time : Three hours
Answer ALL questions
Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

1. State two properties of unit impulse function.

2. Draw the following signals:
   (a) \( u(t) - u(t - 10) \)
   (b) \( (1/2)^n u(n - 1) \).

3. State the conditions for the convergence of Fourier series representation of continuous time periodic signals.

4. Find the ROC of the Laplace transform of \( x(t) = u(t) \).

5. Draw the block diagram of the LTI system described by \( \frac{d^2 y(t)}{dt^2} + y(t) = 0.1 x(t) \).

6. Find \( y(n) = x(n - 1) \ast \delta(n - 2) \).

7. Find the DTFT of \( x(n) = \delta(n) + \delta(n - 1) \).

8. State and prove the time folding property of z-transform.

9. Give the impulse response of a linear time invariant time as \( h(n) = \sin \pi n \), check whether the system is stable or not.

10. In terms of ROC, state the condition for an LTI discrete time system to be causal and stable.
PART B — (5 x 16 = 80 marks)

11. (a) Check whether the following signals are periodic/aperiodic signals.
   (i) \( x(t) = \cos 2t + \sin t/5 \).
   (ii) \( x[n] = 3 + \cos \pi/2n + \cos 2n \).

   Or

(b) Check whether the following system is linear, causal time invariant
and/or stable
   (i) \( y[n] = x[n] - x[n-1] \)
   (ii) \( y(t) = \frac{d}{dt} x(t) \).

12. (a) Find the Fourier series coefficients of the following signal:

```
C
```

   Plot the spectrum of the signal.

   Or

(b) Find the spectrum of \( x(t) = e^{-2t} \). Plot the spectrum of the signal.

13. (a) Find the overall impulse response of the following system.

```
C
```

Here
\( h_1(t) = e^{-2t}u(t) \)
\( h_2(t) = \delta(t) - \delta(t-1) \)
\( h_3(t) = \delta(t) \)

Also find the output of the system for the input \( x(t) = u(t) \) using
convolution integral.

Or
(b) An LTI system is represented by \( \frac{d^2}{dt^2} y(t) + 4 \frac{d}{dt} y(t) + 4 y(t) = x(t) \) with initial conditions \( y(0) = 0; y'(0) = 1 \); Find the output of the system, when the input is \( x(t) = e^{-t} u(t) \).

14. (a) State and prove sampling theorem for a band limited signal.

    Or

(b) Find inverse z-transform of \( X(z) = \frac{z^{-1}}{1 - 0.25z^{-1} - 0.375z^{-2}} \).

For

(i) ROC \( |z| > 0.75 \)

(ii) ROC \( |z| < 0.5 \)

15. (a) Compute \( y(n) = x(n) * h(n) \)

where \( x(n) = (1/2)^n u(n-2) \)

\( h(n) = u(n-2) \).

    Or

(b) LTI discrete time system \( y(n) = 3/2 y(n-1) - 1/2 y(n-2) + x(n) + x(n-1) \) is given an input \( x(n) = u(n) \)

(i) Find the transfer function of the system.

(ii) Find the impulse response of the system.
Question Paper Code : 31354


Third Semester

Electronics and Communication Engineering

EC 2204/EC 35/EC 1202 A/080290015/10144 EC 305 — SIGNALS AND SYSTEMS

(Regulation 2008/2010).

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Give the mathematical and graphical representation of continuous time and
   discrete time unit impulse function.
2. What are the conditions for a system to be LTI system?
3. State Dirichlet's conditions.
4. Give the equation for trigonometric Fourier series.
5. What are the three elementary operations in block diagram representation of
   continuous time system?
6. Check whether the causal system with transfer function \( H(s) = \frac{1}{s-2} \) is stable.
7. What is aliasing?
8. Define unilateral and bilateral Z transform.
9. Define convolution sum with its equation.
10. Check whether the system with system function \( H(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{1}{1 - 2z^{-1}} \)
    with ROC \( |Z| < \infty \) is causal and stable.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Determine whether the signal \( x(t) = \sin 20 \pi t + \sin 5 \pi t \) is periodic
    and if it is periodic find the fundamental period. (5)
    (ii) Define energy and power signals. Find whether the signal
         \( x(n) = \left( \frac{1}{2} \right)^n u(n) \) is energy or power signal and calculate their
         energy or power. (5)
    (iii) Discuss various forms of real and complex exponential signals with
         graphical representation. (6)

Or
(b) Determine whether the discrete time system \( y(n) = x(n)\cos(\omega n) \) is

(i) Memoryless  (ii) Stable

(iii) Causal  (iv) Linear

(v) Time invariant.

12. (a) (i) Find the exponential Fourier series of the waveform

(ii) Find the Fourier transform of the signal \( x(t) = e^{-|t|} \).

Or

(b) (i) Find the Laplace transform of the signal \( f(t) = e^{-|t|}\sin \omega t \).

(ii) Find the inverse Fourier transform of the rectangular spectrum given by

\[
X(j\omega) = \begin{cases} 
1, & -W < \omega < W \\
0, & |\omega| > W 
\end{cases}
\]

13. (a) (i) Define convolution integral and derive its equation.

(ii) A stable LTI system is characterized by the differential equation

\[
\frac{d^2y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)
\]

Find the frequency response and impulse response using Fourier transform.

Or

2 31354
(b) (i) Draw direct form, cascade form and parallel form of a system with system function.

\[ H(s) = \frac{1}{(s+1)(s+2)} \]  \hspace{1cm} (8)

(ii) Determine the state variable description corresponding to the block diagram given below.

14. (a) (i) Determine the discrete time Fourier transform of \( x(n) = d^{-n}, |d| < 1 \).

(ii) Find the \( z \)-transform and ROC of the sequence \( x(n) = r^n \cos(n \theta) u(n) \).

Or

(b) (i) State and prove the following properties of \( z \)-transform

1. Linearity
2. Time shifting
3. Differentiation
4. Correlation.

(ii) Find the inverse \( z \)-transform of the function

\[ X(z) = \frac{1 + z^{-1}}{\left(1 - \frac{2}{3} z^{-1}\right)^2}, \text{ROC } |z| > \frac{2}{3} \]  \hspace{1cm} (8)
15. (a) (i) Compute convolution sum of the following sequences

\[ x(n) = \begin{cases} 
1, & 0 \leq n \leq 4 \\
0, & Otherwise 
\end{cases} \]

and

\[ h(n) = \begin{cases} 
\alpha^n, & 0 \leq n \leq 6 \\
0, & Otherwise 
\end{cases} \] \hspace{2cm} (10)

(ii) Draw direct form I and direct form II implementations of the system described by difference equation.

\[ y(n) + \frac{1}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) + x(n-1). \] \hspace{2cm} (6)

Or

(b) (i) Determine the transfer function and the impulse response for the causal LTI system described by the difference equation using \( z \) transform.

\[ y(n) - \frac{1}{4} y(n-1) - \frac{3}{8} y(n-2) = -x(n) + 2x(n-1). \] \hspace{2cm} (8)

(ii) Develop the state variable description for the discrete time system given below.

![Diagram of the system](image-url)
Question Paper Code: 51396

Third Semester
Electronics and Communication Engineering
EC 2204/EC 35/EC 1202 A/08290015/10144 EC 305 – SIGNALS AND SYSTEMS
(Regulation 2008/2010)

Time: Three hours
Maximum: 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Sketch the following signals
   (a) \( x(t) = 2t \) for all \( t \)
   (b) \( x(n) = 2n - 3 \) for all \( n \)

2. Given \( x[n] = \{1, -4, 3, 1, 5, 2\} \). Represent \( x[n] \) in terms of weighted shifted impulse functions.

3. State the conditions for convergence of fourier series.

4. State any two properties of ROC of Laplace transform \( X(s) \) of a signal \( x(t) \).

5. State the necessary and sufficient condition for an LTI continuous time system to be Causal.

6. Find the differential equation relating the input and output of a CT system represented by \( H(j\Omega) = \frac{4}{(j\Omega)^3 + 8j\Omega + 4} \)

7. What is an anti-aliasing filter?

8. State the multiplication property of DTFT.
9. Find the overall impulse response $h(n)$ when two systems \( h_1(n) = u(n) \) and
\( h_2(n) = \delta(n) + 2\delta(n-1) \) are in series.

10. Using Z-transform, check whether the following system is stable.
\[
H(z) = \frac{z}{z - \frac{1}{2}} + \frac{2z}{z - 3}, \quad \frac{1}{2} < |z| < 3.
\]

PART B — \((5 \times 16 = 80 \text{ marks})\)

11. (a) (i) Given \( x(t) = \begin{cases} \frac{1}{6}(t+2), & -2 \leq t \leq 4 \\ 0, & \text{otherwise} \end{cases} \)
Sketch (1) \( x(t) \) (2) \( x(t+1) \) (3) \( x(2t) \) (4) \( x(t/2) \).

(ii) Determine whether the discrete time sequence
\[
x[n] = \sin \left( \frac{3\pi}{7} n + \frac{\pi}{4} \right) + \cos \frac{\pi}{3} n
\]
is periodic or not.

Or

(b) Check the following systems are linear, stable
(i) \( y(t) = e^{at} \)
(ii) \( y(n) = x(n-1) \).

12. (a) Find the fourier series coefficients of the signal shown below:

(b) Find the inverse laplace transform of \( X(s) = \frac{1}{(s + 5)(s - 3)} \) for the ROCs
\[
\text{i) } -5 < \text{Re}(s) < 3.
\]
\[
\text{ii) } \text{Re}(s) > 3
\]
13. (a) Using convolution integral, determine the response of a CT-LTI system \( y(t) \) given input \( x(t) = e^{-\alpha t}u(t) \) and impulse response \( h(t) = e^{-\beta t}u(t) \), \( |\alpha| < 1 \), \( |\beta| < 1 \).

Or

(b) Find the frequency response of the system shown below:

14. (a) Using convolution property of DTFT, find the inverse DTFT of

\[
X(e^{j\omega}) = \frac{1}{(1 - a e^{-j\omega})^2}, \quad |a| < 1.
\]

Or

(b) Find the inverse Z-transform of \( X(z) = \frac{z^2}{(z - 0.5)(z - 1)}, \quad |z| > 1 \).

15. (a) Find the convolution of sum of \( x[n] = r[n] \) and \( h[n] = u[n] \).

Or

(b) A causal LTI system is described by

\[
y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]
\]

where \( x[n] \) is the input to the system \( h[n] \) is the impulse response of the system. Find

(i) System function \( H(z) \)

(ii) Impulse response \( h(n) \).